Axion detection with spin dynamics: magnons and axions

SC, Takeo Moroi, Kazunori Nakayama

arXiv: 2001.10666, 2102.06179

So Chigusa

4/6/2023 @ Joint IQ Initiative & PITT PACC Workshop

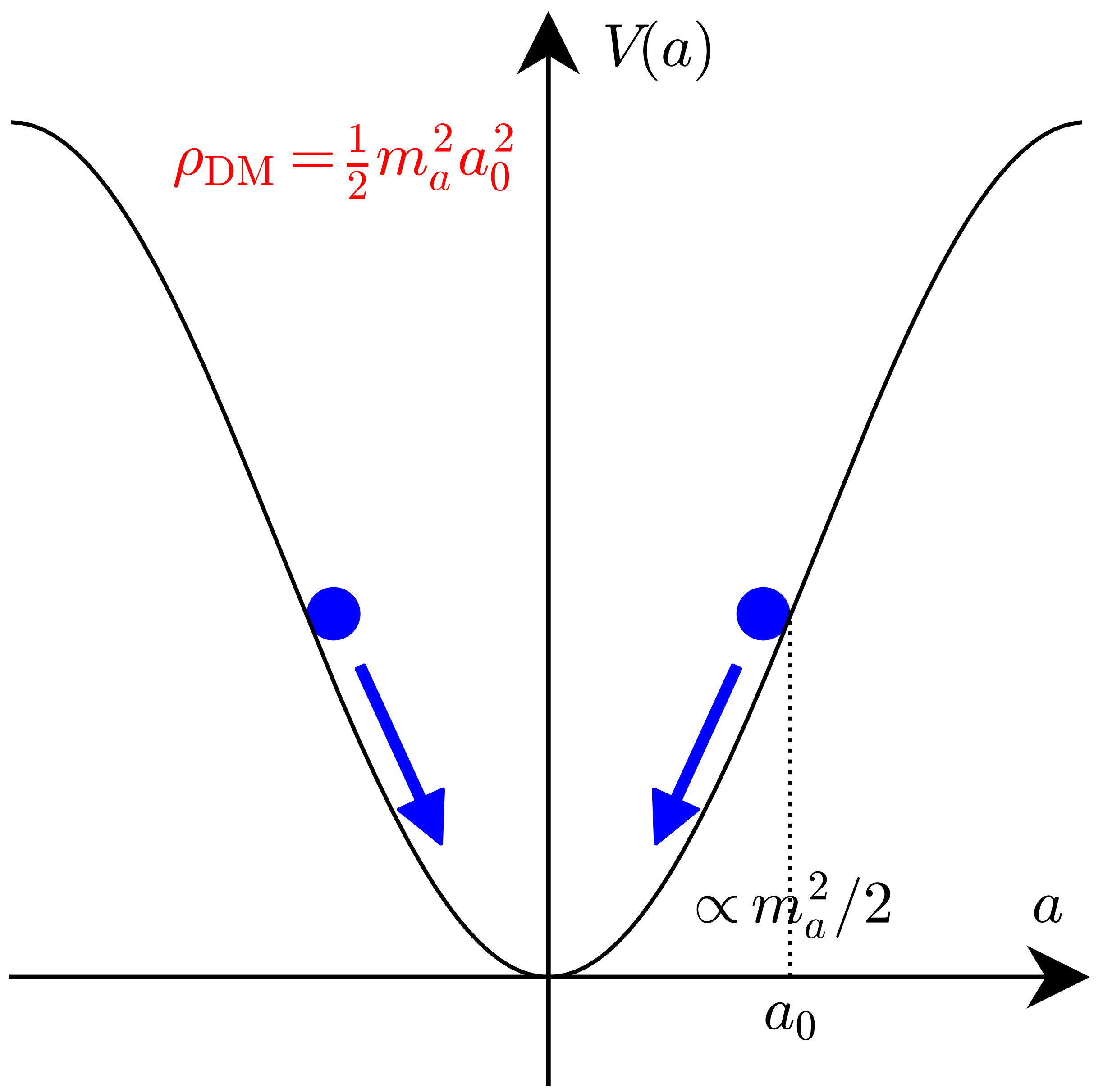
Axion dark matter

- QCD axion is highly motivated by the strong CP problem
- Axion-like particles (ALPs) are motivated by the string theory

Arvanitaki et al., (2009)

- Light axion can explain the DM relic abundance through the misalignment mechanism

$$a(\vec{x}, t) = a_0 \cos(m_a t + \delta)$$



- Has model-dependent interactions with fermions

$$\mathscr{L} = g_{aff} \frac{\partial_\mu a}{2m_f} \bar{f} \gamma^\mu \gamma_5 f \quad \rightarrow \quad H_{\rm eff} = \frac{g_{aff}}{m_f} \nabla a \cdot \mathbf{S}_f$$

  - These interactions work as effective magnetic fields $\vec{B}_a^{(f)} \sim \sqrt{2\rho_{\rm DM}} \frac{g_{aff}}{e} \vec{v}_{\rm DM} \sin(m_a t + \delta)$ that couple to the fermion spins

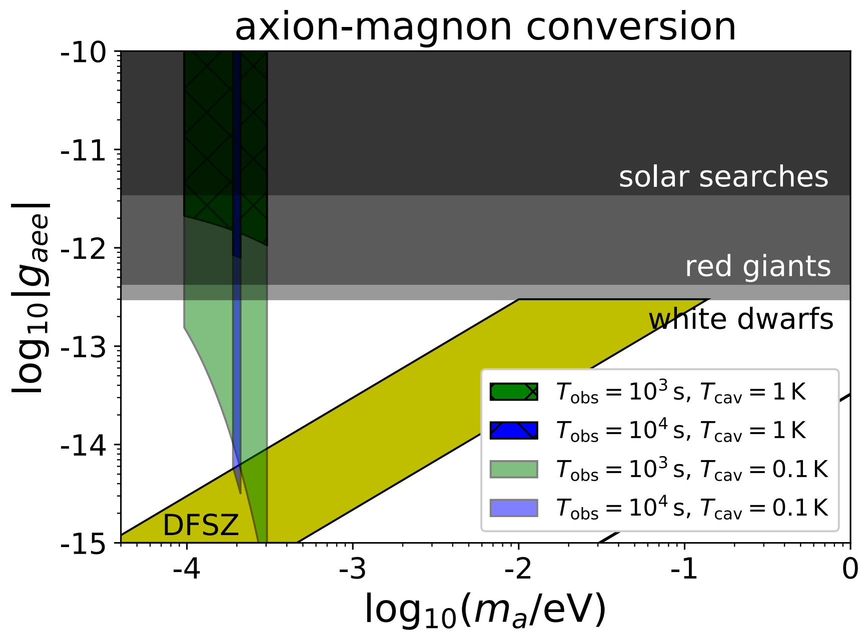
Spin dynamics for axion DM search

- Various setups are considered depending on the coupling/mass range of interest

Electron spins

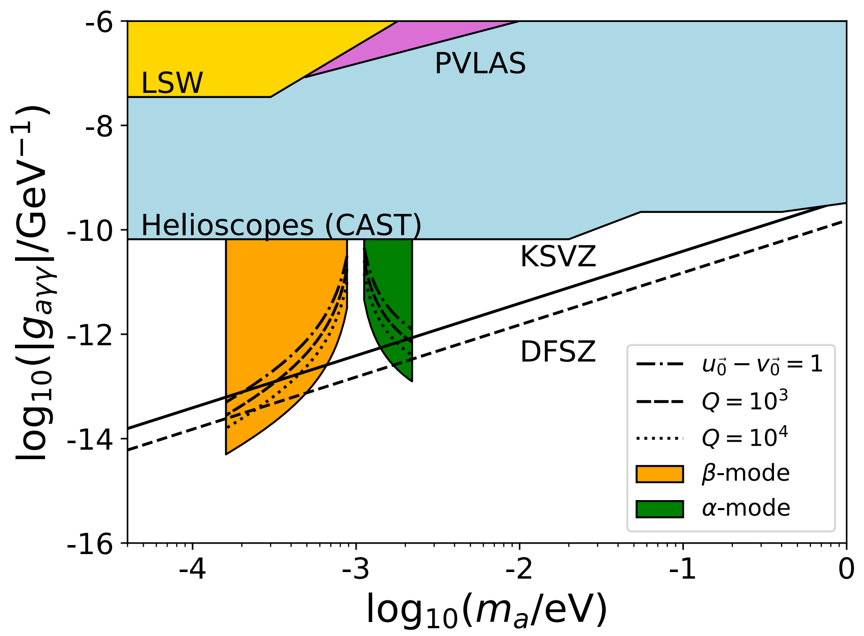
Today's topics

- Magnons: g_{aee}



2001.10666

- Axions: $g_{a\gamma\gamma}$



2102.06179

- Application of the NV center magnetometry with diamond samples
  - g_{aee} , $m_a \lesssim 10^{-4}\,\mathrm{eV}$

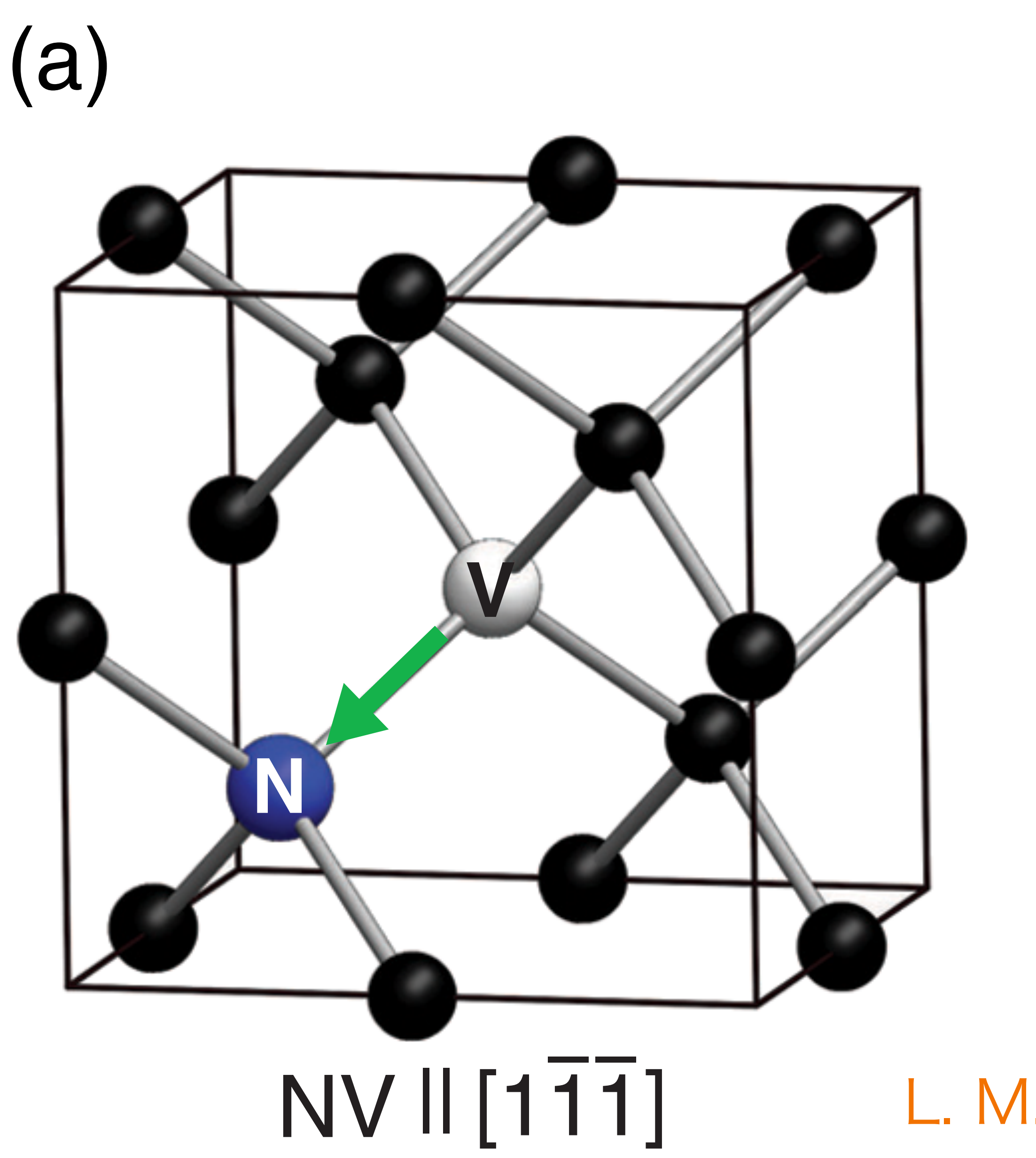
2302.12756

Nuclear spins

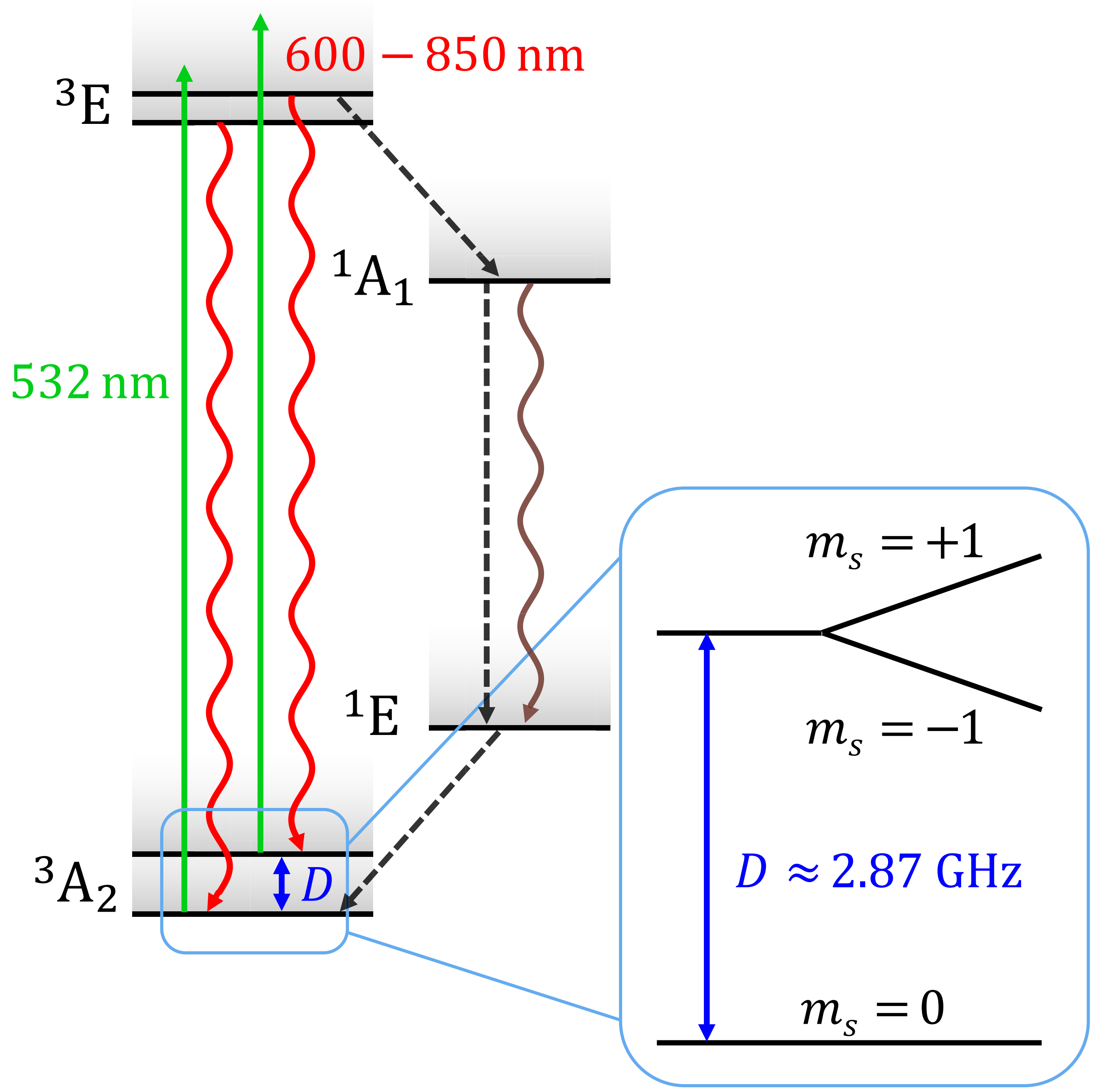
- The ferromagnetic phase of superfluid $^3\mathrm{H_e}$
  - g_{ann} , $m_a \sim 10^{-6}\,\mathrm{eV}$ in preparation
- Materials with strong hyperfine interaction
  - g_{ann} , $10^{-6}\,\mathrm{eV} \lesssim m_a \lesssim 10^{-4}\,\mathrm{eV}$ in preparation

Only my works are shown just as examples

Brief comment on the NV center



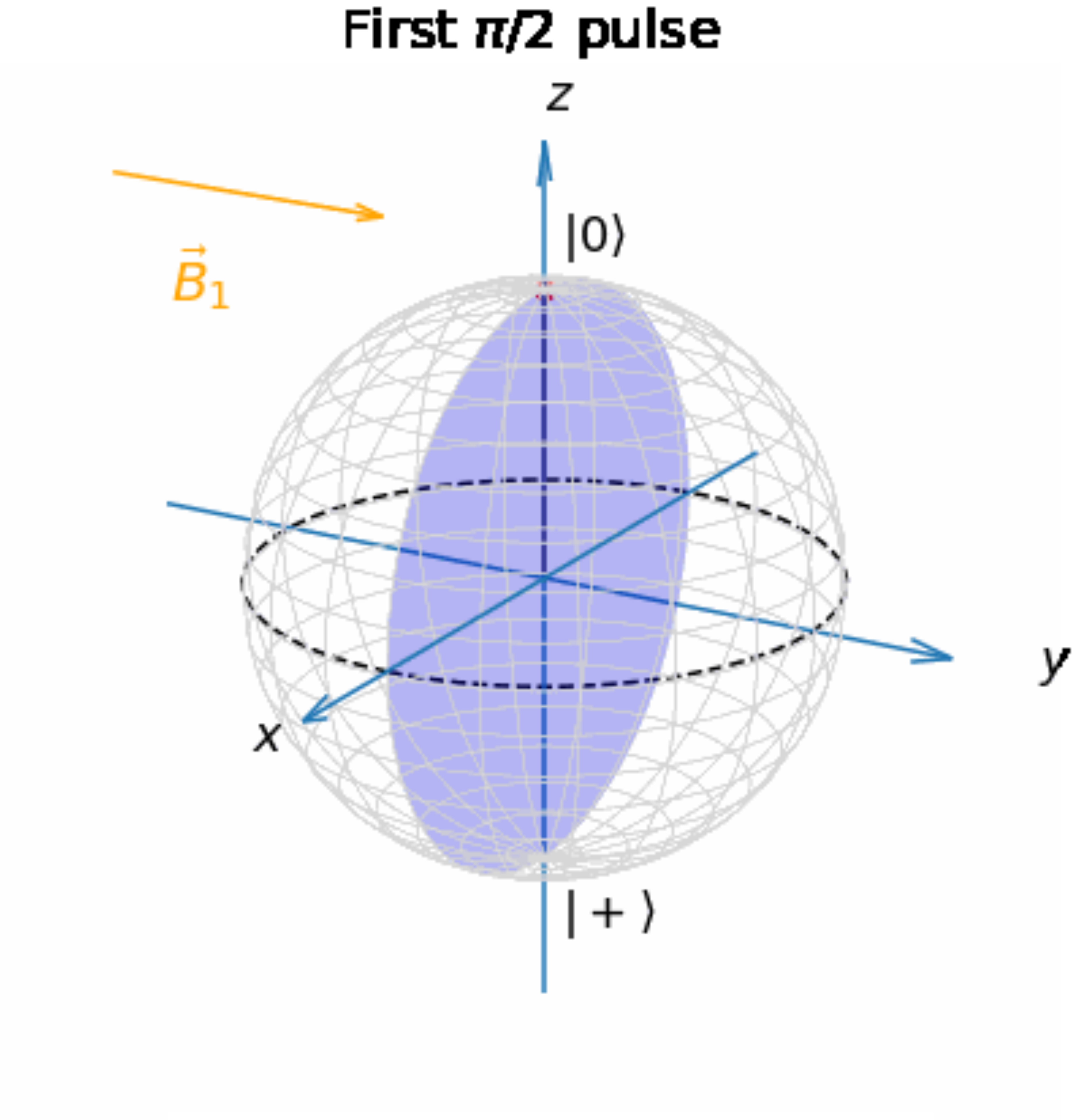
L. M. Pham '13

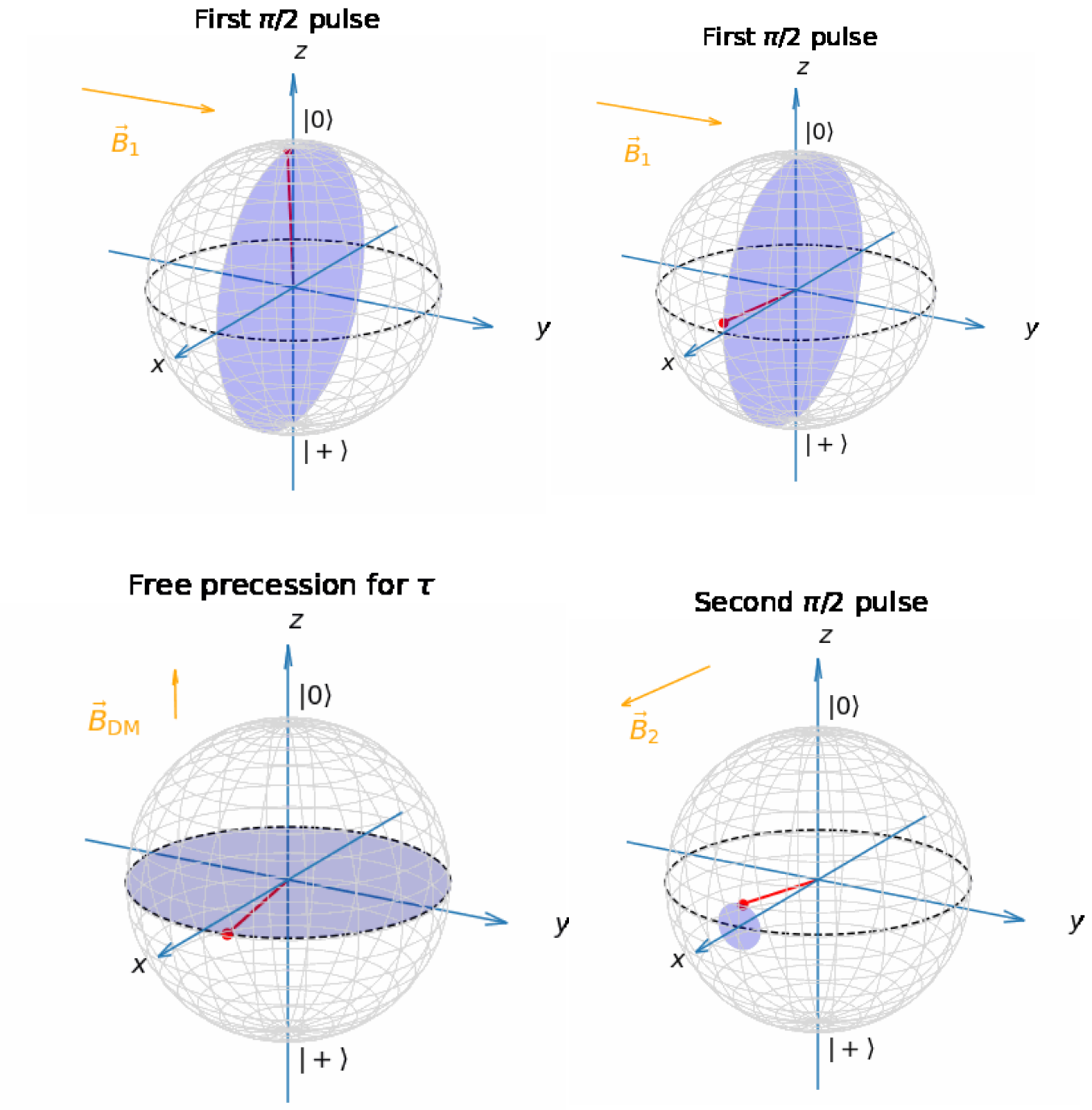


J. F. Barry+ '20

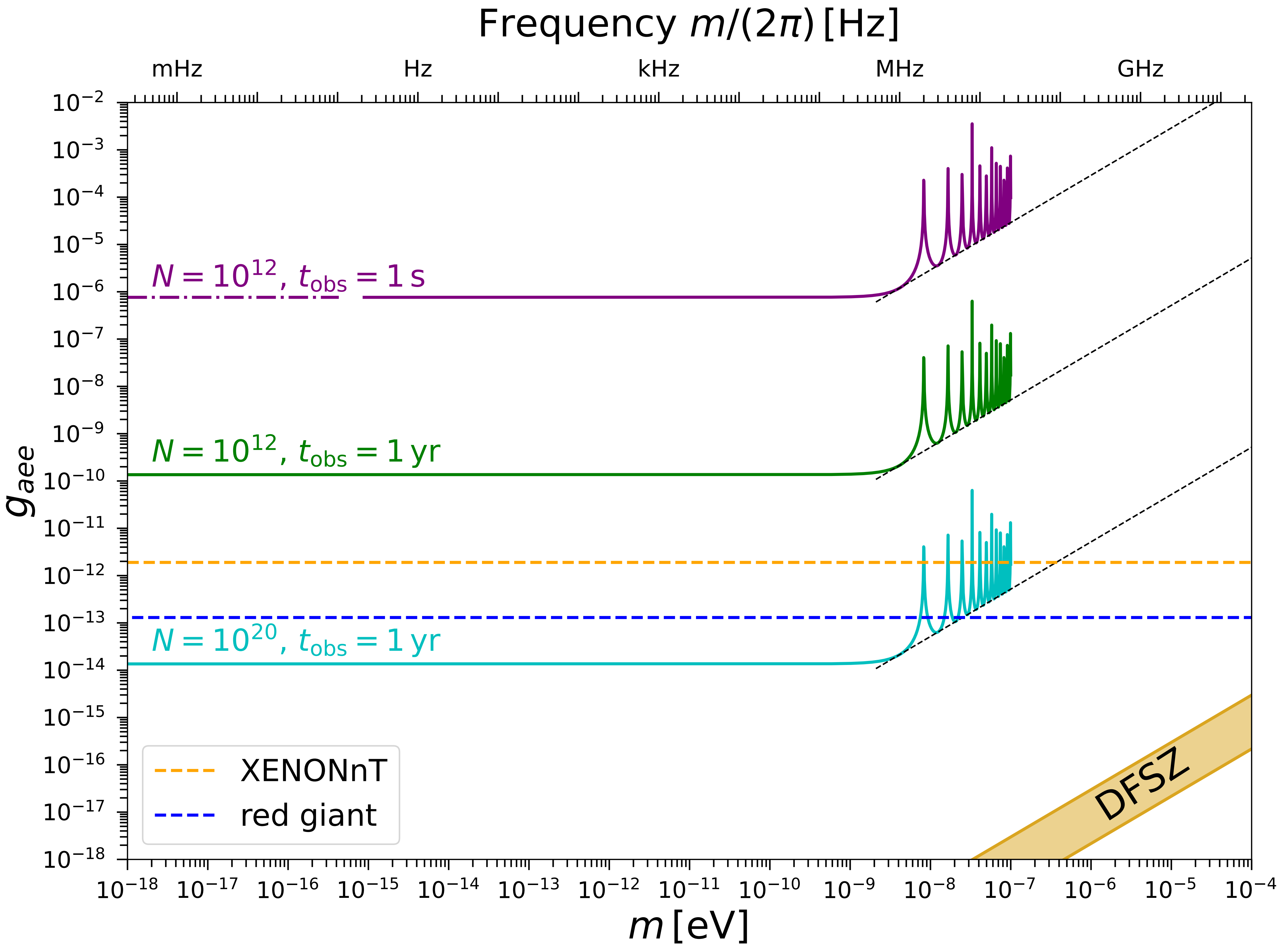
- The NV center in diamond hosts an e^- spin triplet system
- Fluorescence enables us to measure the quantum state of the e^- spin system
- The NV center works as a quantum sensor of, for example, the magnetic field

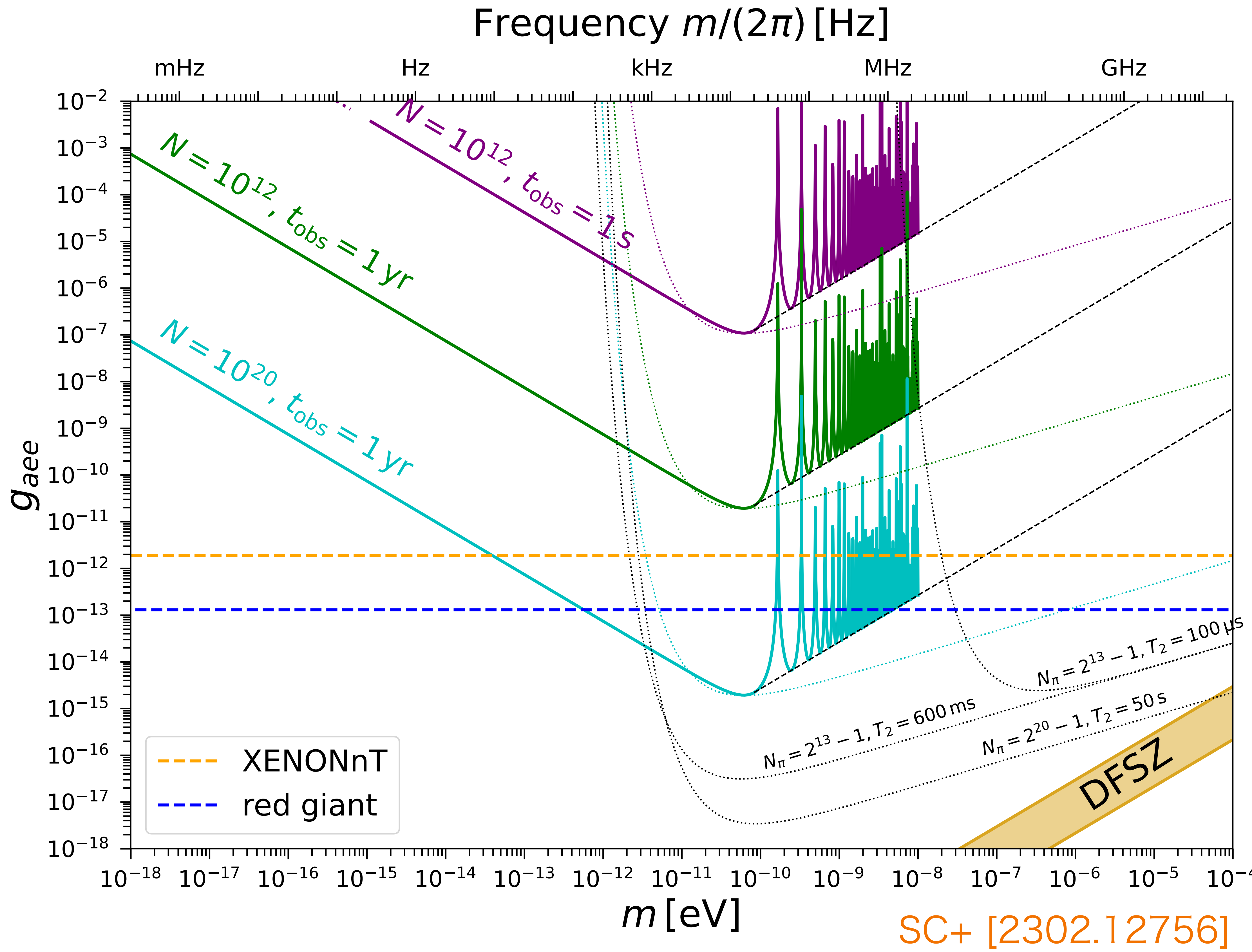
- The NV center dc magnetometry





The NV center sensitivity on axion DM





SC+ [2302.12756]

- Sensitivity on g_{aee} for a broad mass range $m_a \lesssim 10^{-4}\,\mathrm{eV}$

Magnons

We need collective excitations

- Sub-MeV DM has a small momentum transfer $q \ll \mathrm{keV}$
  - DM de Broglie wavelength is longer than the interatomic distance ~ a few $\mathrm{\AA}$

$$\lambda_{\text{de Broglie}} \sim 1\,\mathrm{m}\left(\frac{10^{-4}\,\mathrm{eV}}{m_{\mathrm{DM}}}\right)$$

  - Axion effectively works as a spatially uniform magnetic field

- DM excites the collective motion of spins rather than individual spin

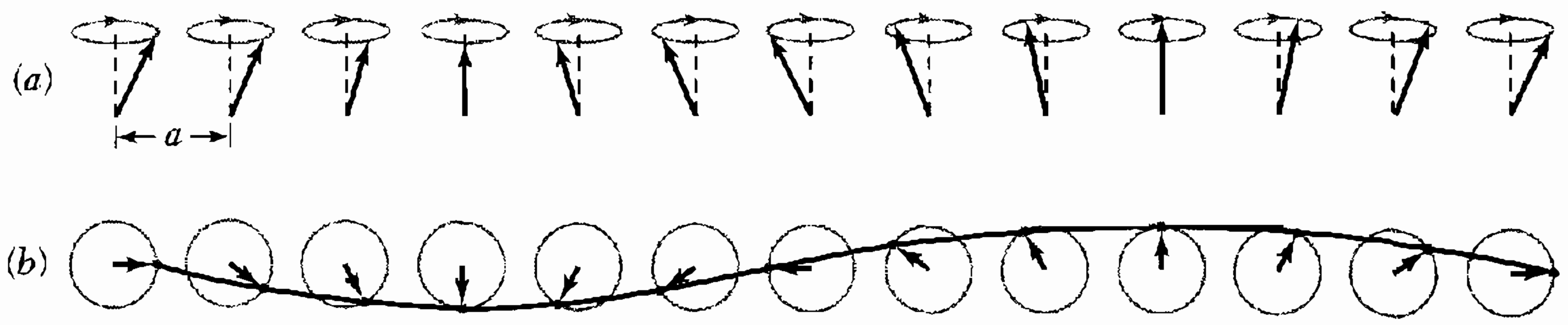


Figure 9 A spin wave on a line of spins. (a) The spins viewed in perspective. (b) Spins viewed from above, showing one wavelength. The wave is drawn through the ends of the spin vectors.

C. Kittel "Introduction to Solid State Physics [8th ed]"

(Ferromagnetic) magnon properties

- Magnon is a bosonic quasi-particle corresponding to the spin wave excitation

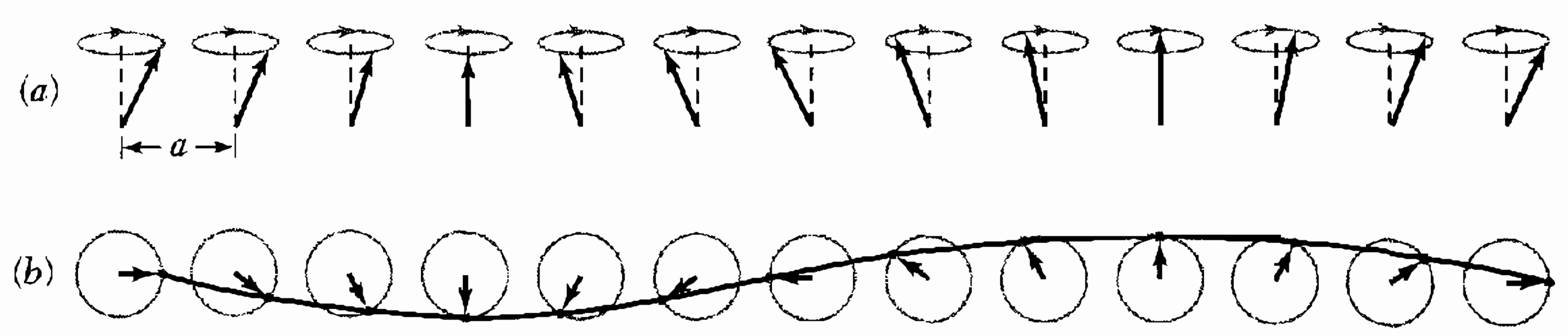


Figure 9 A spin wave on a line of spins. (a) The spins viewed in perspective. (b) Spins viewed from above, showing one wavelength. The wave is drawn through the ends of the spin vectors.

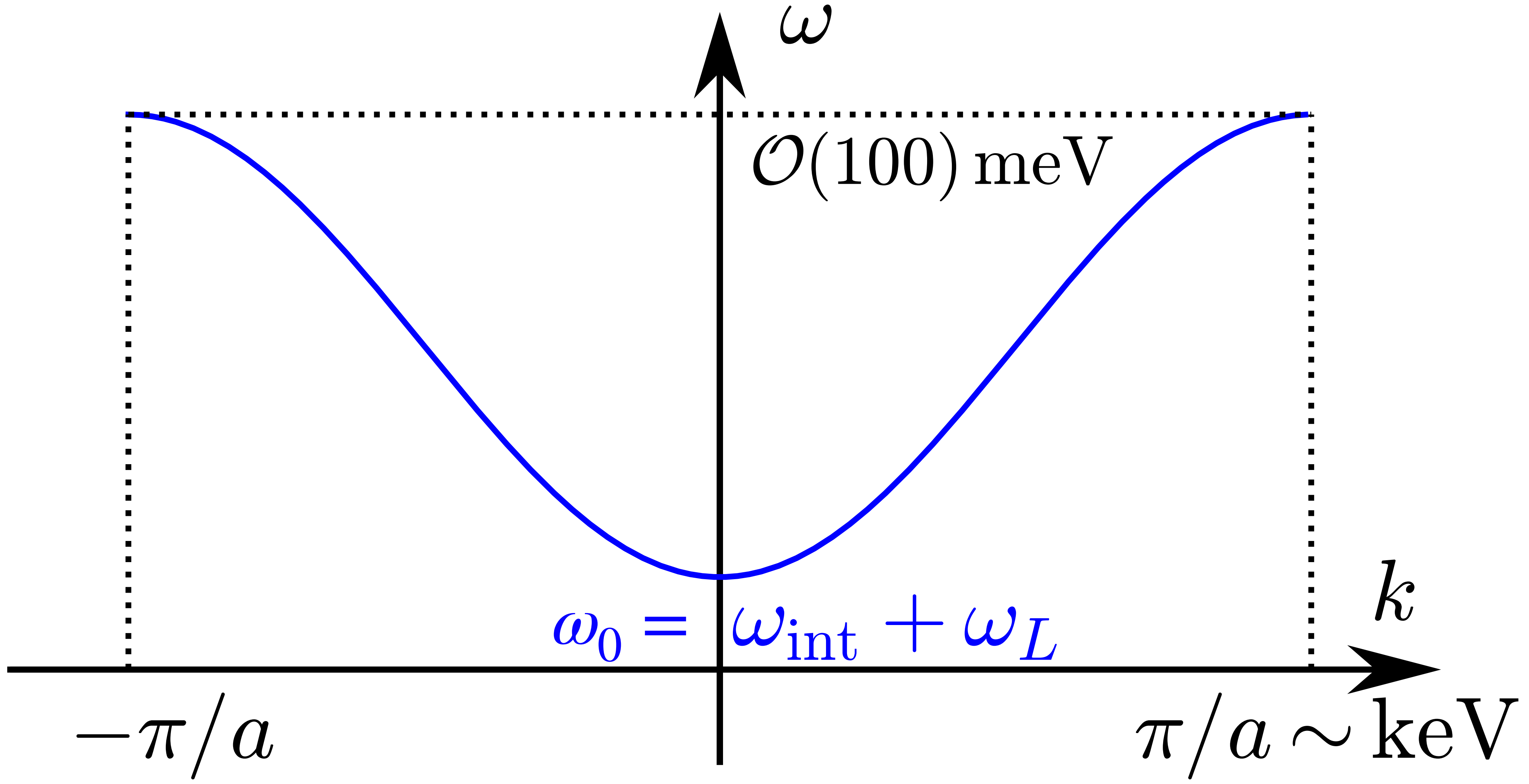
C. Kittel "Introduction to Solid State Physics [8th ed]"

- Typical scales
  - momentum $k \lesssim \mathrm{keV}$
  - energy $\omega \lesssim \mathcal{O}(100)\,\mathrm{meV}$

- Magnon is a NGB of spin $\mathrm{SO}(3)$ rotation
- Gapped due to soft breaking of $SO(3)$
  - Anisotropy of the crystal $\omega_{\mathrm{int}} \sim 0 - 100\,\mathrm{meV}$
  - External magnetic field ω_L

Mitridate, et al. '20

$$\omega_L \sim 0.12\,\mathrm{meV}\left(\frac{B_0}{1\,\mathrm{T}}\right)$$

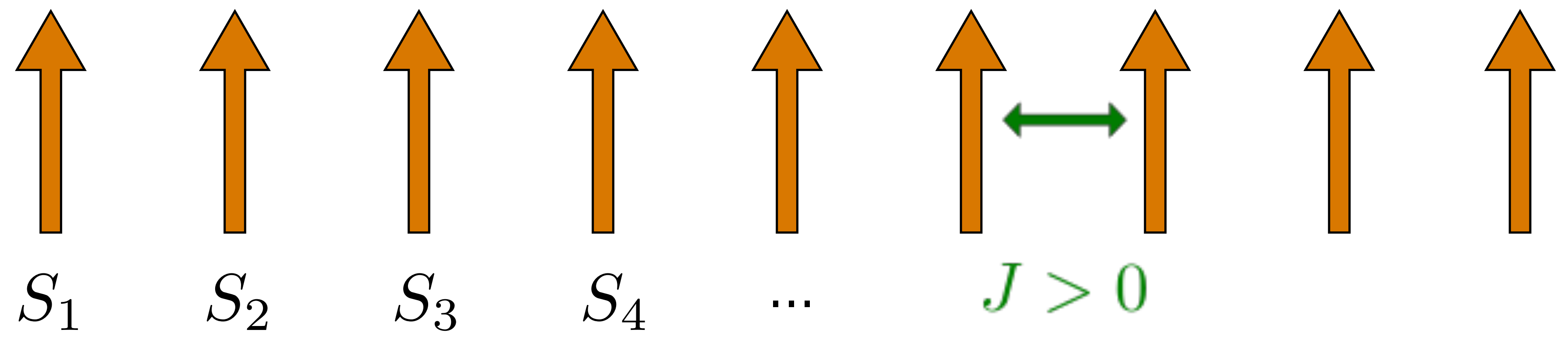


Quantum description of magnon

- Start with a 1D ferromagnetic system of spin-s

$$H = -J \sum_{\ell} \vec{S}_{\ell} \cdot \vec{S}_{\ell+1} - g_e \mu_B B_0 \sum_{\ell} S^z_{\ell}$$

with $J > 0$



- Express spin fluctuations with bosonic operators a_ℓ

$$S^+_\ell \equiv S^x_\ell + iS^y_\ell = \sqrt{2s}\sqrt{1 - \frac{a^\dagger_\ell a_\ell}{2s}}\, a_\ell$$

$$S^-_\ell \equiv S^x_\ell - iS^y_\ell = \sqrt{2s}\, a^\dagger_\ell \sqrt{1 - \frac{a^\dagger_\ell a_\ell}{2s}}$$

$$S^z_\ell = s - a^\dagger_\ell a_\ell$$

- Commutation relations are consistent:

$$\left[S^i_\ell, S^j_\ell\right] = i\epsilon^{ijk} S^k_\ell \Leftrightarrow \left[a_\ell, a^\dagger_\ell\right] = 1$$

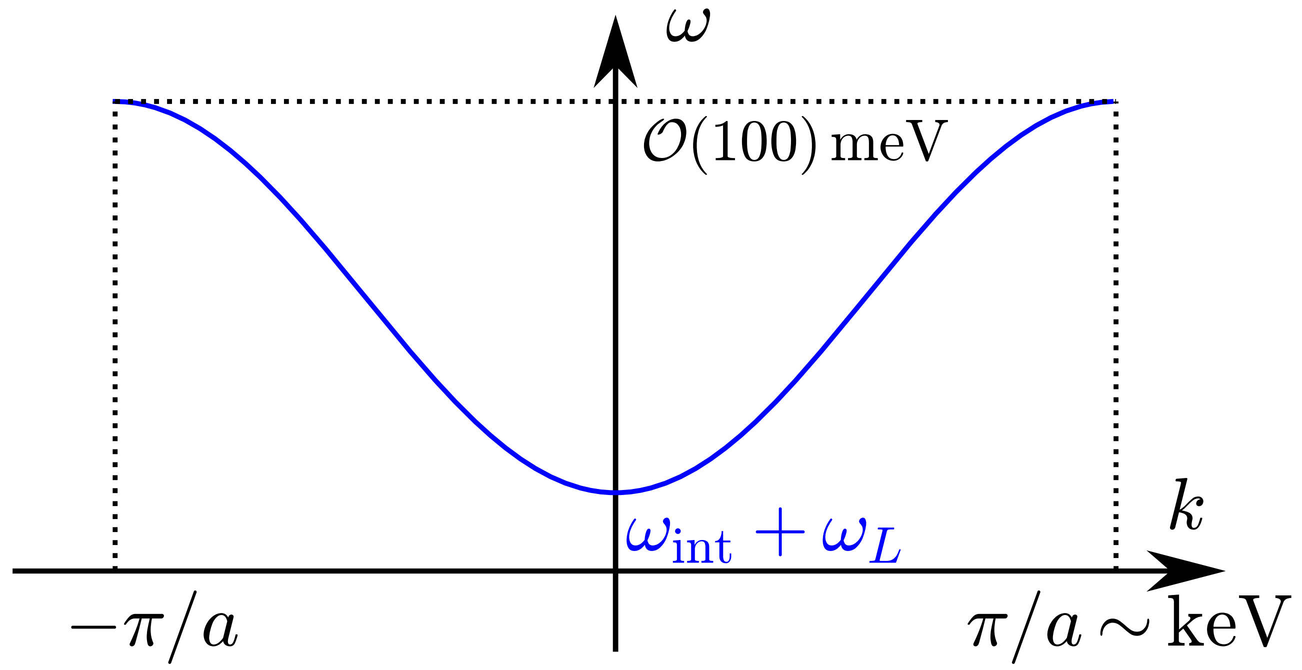
- Using Fourier transformation, we obtain

$$H \simeq \sum_k \big[\underbrace{2Js(1-\cos(ka))}_{\omega_k} + \underbrace{g_e \mu_B B_0}_{\omega_L}\big]\, a^\dagger_k a_k + \cdots$$

- Also, we can check

$$\sum_\ell S^z_\ell = Ns - \sum_k a^\dagger_k a_k$$

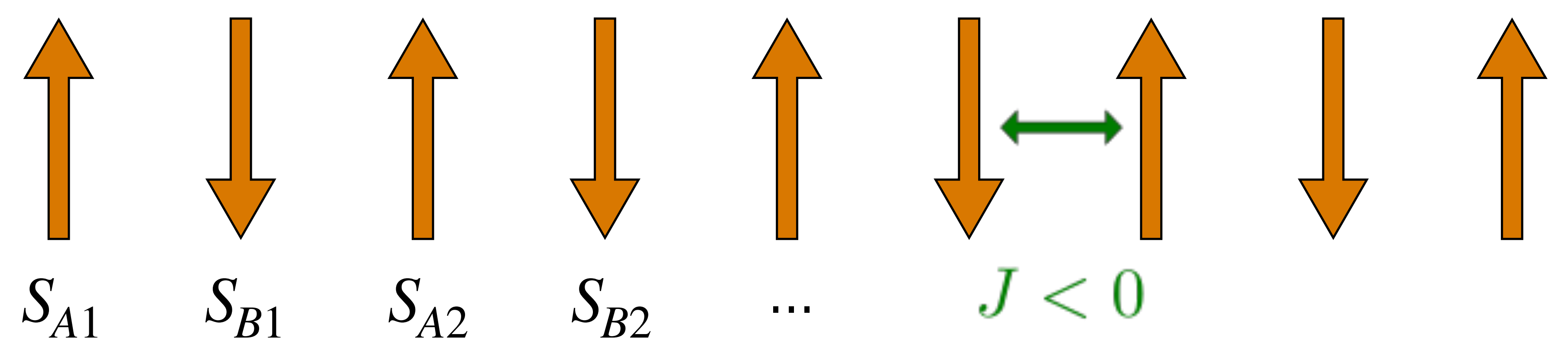
A magnon has $\Delta S^z = -1$



Brief comment on anti-ferromagnet

- Consider instead a 1D anti-ferromagnetic system of spin-s

  $$H = -J \sum_{\ell} \vec{S}_{\ell} \cdot \vec{S}_{\ell+1} \text{ with } J < 0$$



- Two sub-lattices are treated differently

  $$S^+_{A\ell} \simeq \sqrt{2s}a_\ell \ ; \ S^-_{A\ell} \simeq \sqrt{2s}a^\dagger_\ell \ ; \ S^z_{A\ell} = s - a^\dagger_\ell a_\ell$$

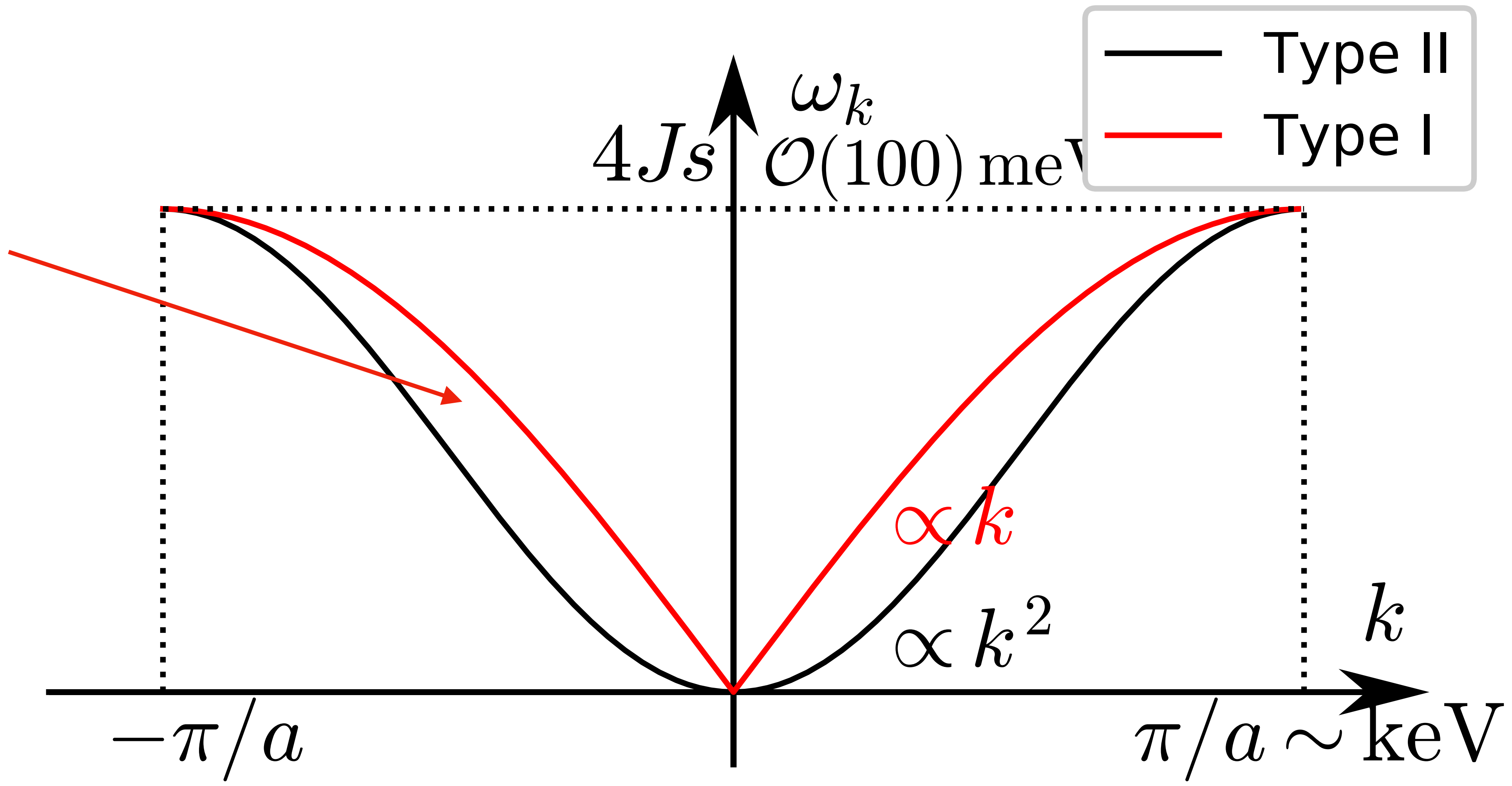
  $$S^+_{B\ell} \simeq \sqrt{2s}b^\dagger_\ell \ ; \ S^-_{B\ell} \simeq \sqrt{2s}b_\ell \ ; \ S^z_{B\ell} = -s + b^\dagger_\ell b_\ell$$

- There are 2 magnon modes with spin ↑ / ↓
  - Classification of non-relativistic NGBs associated with $SO(3) \to SO(2)$ breaking

    Watanabe & Murayama '12, Hidaka '12

  Ferromagnet has 1 Type-II NGB

  Anti-ferromagnet has 2 Type-I NGBs



Sensitivity on axion DM

- DM-magnon conversion in ferromagnetic YIG is described by

$$H_{\rm int} = -g_S \mu_B \vec{S}_e \cdot \vec{B}_a$$

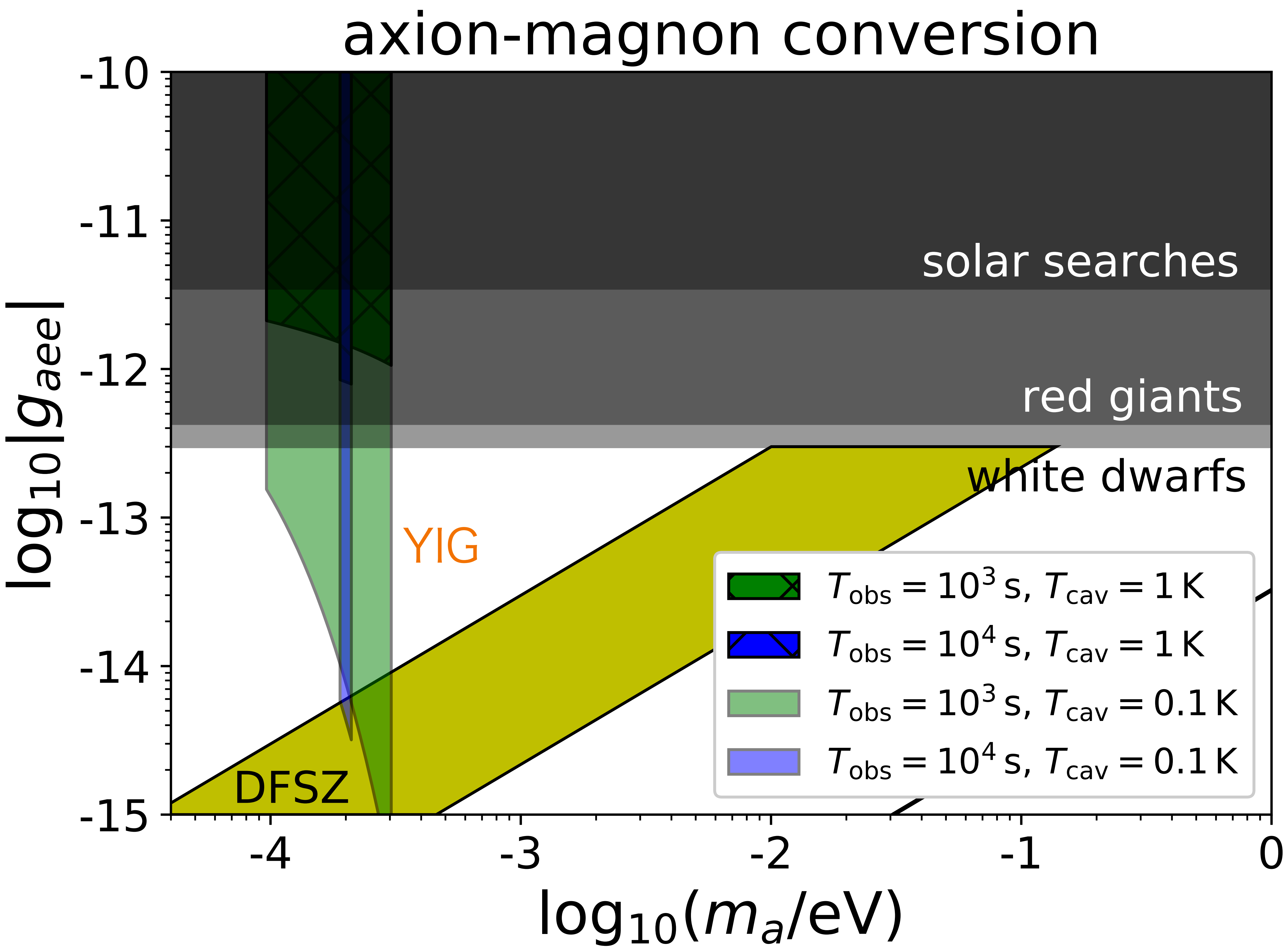
$$= \sin(m_a t + \delta)\left(\sqrt{\frac{sN}{2}}\frac{m_a a_0 v_a^+}{f_a}\tilde{a}_0^\dagger + \text{h.c.}\right)$$

$\tilde{a}_0 : k = 0$ (Kittel) mode of magnon

- Resonance at $\omega_0 = \omega_{\rm int} + \omega_L \simeq m_a$
  - Scan magnetic field $B_0 \sim \mathcal{O}(1)\,{\rm T}$
  - Fixed total observation $T_{\rm total}$
  - Observation time $T_{\rm obs}$ for each scan step

- QUAX experiment

Barbieri, et al. '89, Barbieri, et al. '16, Crescini, et al. '20

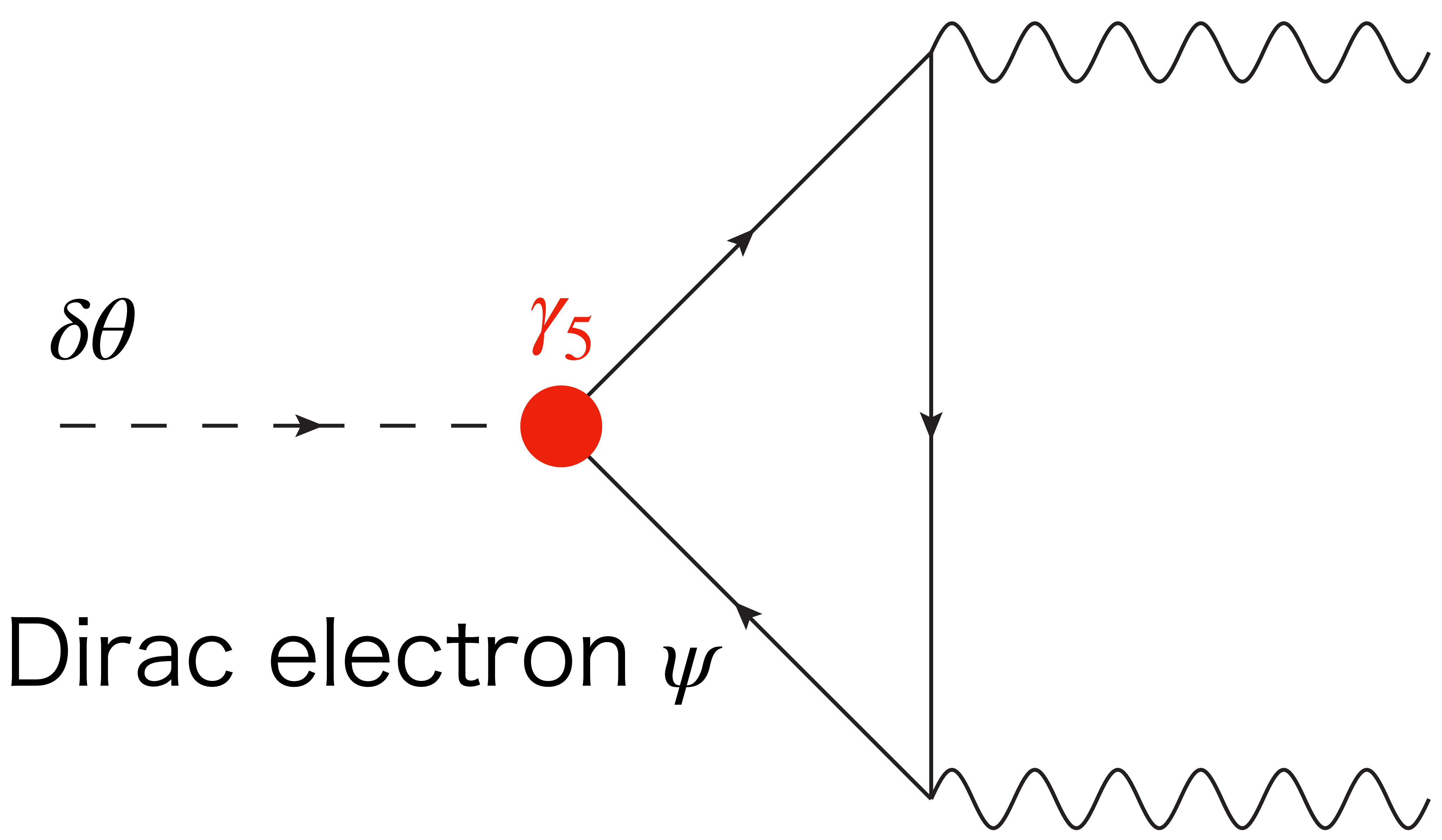


SC, Moroi, Nakayama [2001.10666]

Axions

Axion properties

- Axion is a spin fluctuation $\delta\theta$ in a magnetic material with the following interaction:



$\propto \delta\theta\, F^{\mu\nu}\tilde{F}_{\mu\nu}$

R. Li, J. Wang, X. Qi, S. Zhang Nature Physics 6, 284–288 (2010)

- Examples include the FKM model + Hubbard interaction (a model of anti-ferromagnetic topological insulator)

A. Sekine, K. Nomura '14

The Fu-Kane-Mele (FKM) model

L. Fu, C. L. Kane, E. J. Mele, PRL 98, 106803 (2007)

- Lattice & 1st Brillouin zone structure

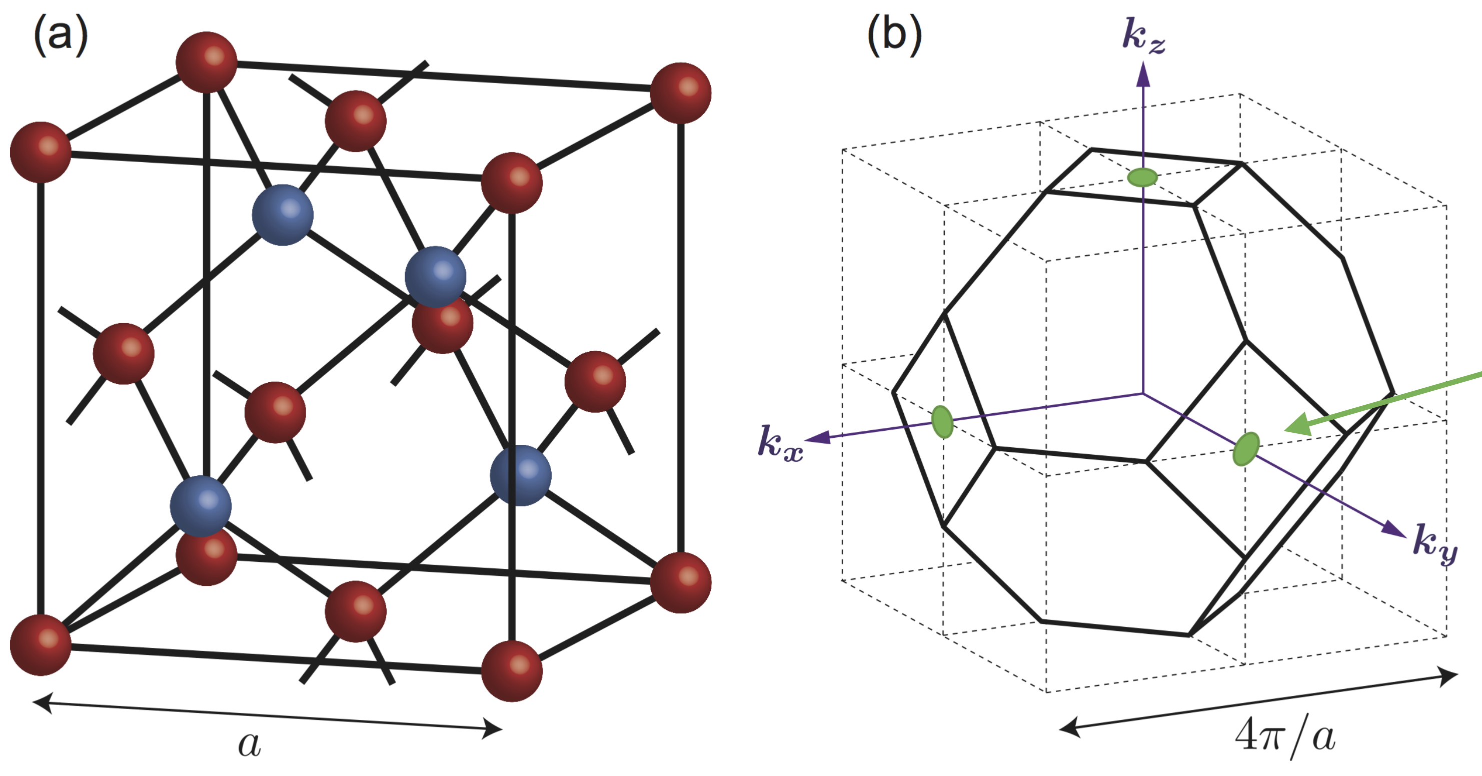
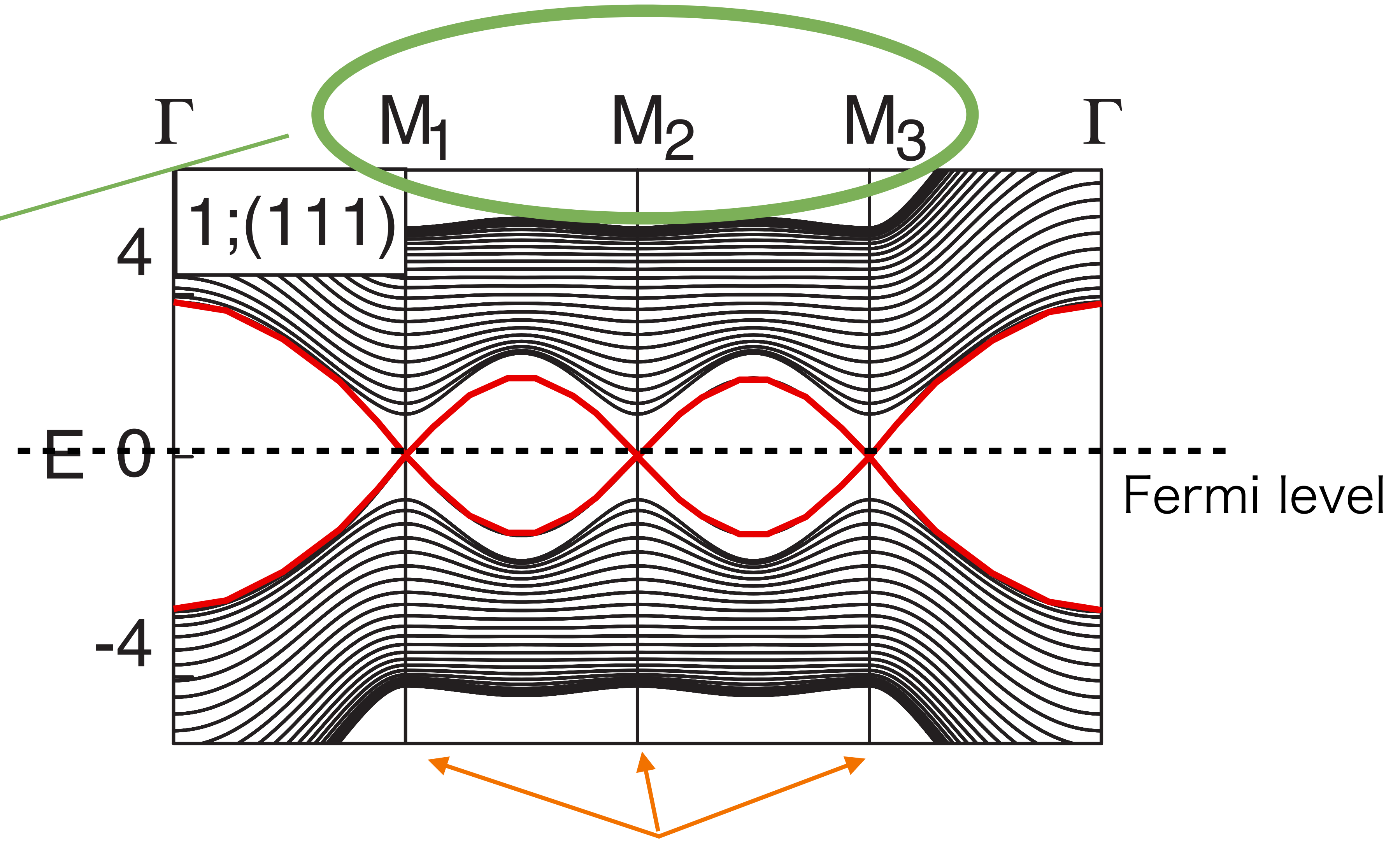


Figure from Sekine, et al. '14

- Band structure



Symmetry-enhanced points

- Hamiltonian of outermost electrons

$$H = -t \sum_{\langle \ell, m \rangle} c_\ell^\dagger c_m + i\lambda \sum_{\langle\langle \ell, m \rangle\rangle} c_\ell^\dagger \vec{\sigma} \cdot (\vec{d}^{\,1}_{\ell m} \times \vec{d}^{\,2}_{\ell m}) c_m$$

- Band crossing occurs at symmetry-enhanced points at the Fermi level, representing its nature as a semimetal

Low-energy effective action

- Low-energy phenomenology is mainly determined by ——— bands
- Those modes around $k = M_r$ are expressed by the effective action

$$S = \int d^4x \sum_{r=1,2,3} \bar{\psi}_r \left[i\gamma^\mu(\partial_\mu - ieA_\mu) - \delta t \right] \psi_r$$

with Dirac electron $\psi \sim (A\uparrow, A\downarrow, B\uparrow, B\downarrow)$

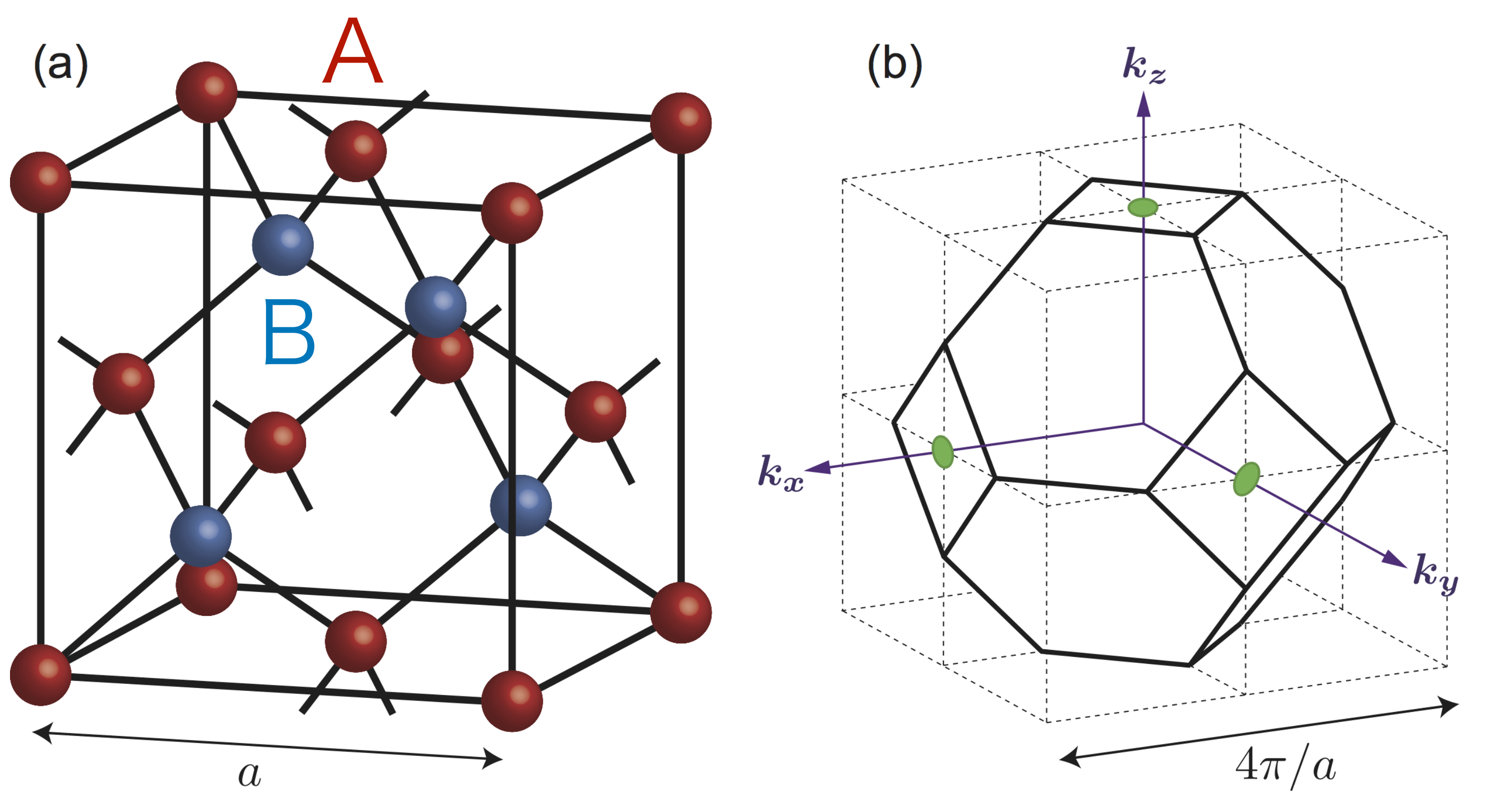
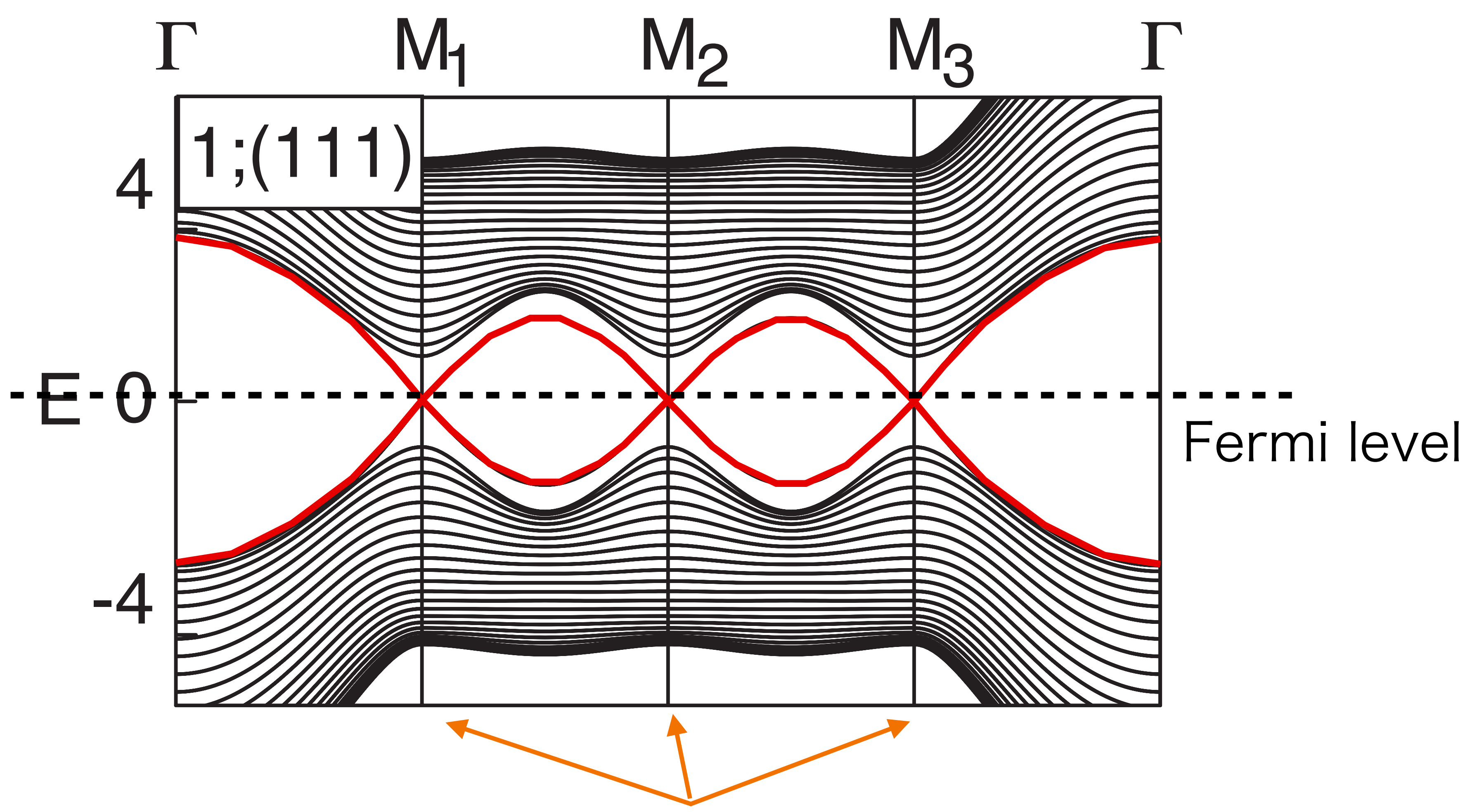


Figure from Sekine, et al. '14



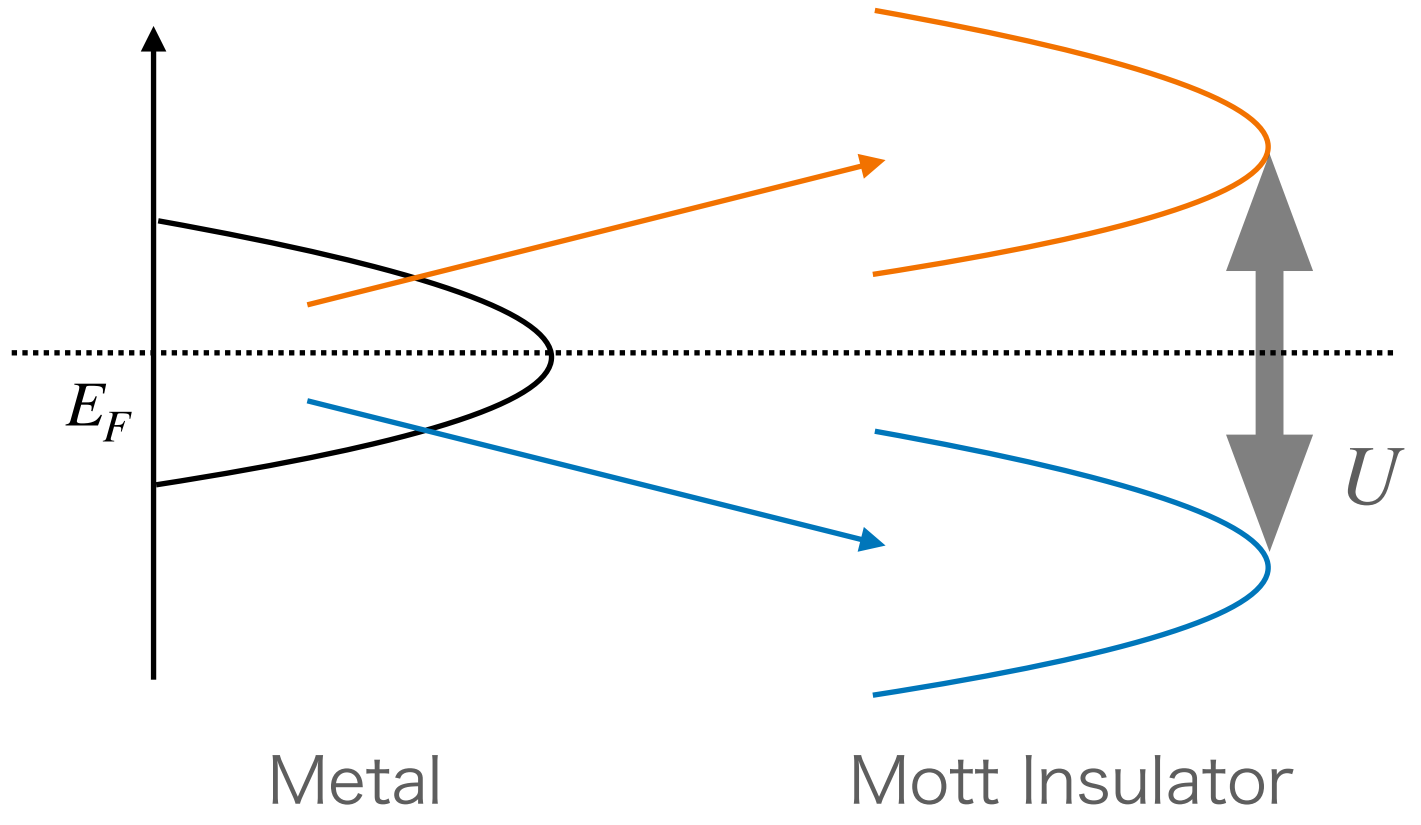
- A small gap δt can be introduced through symmetry breaking (e.g., lattice distortion)

Hubbard interaction and magnetic ordering

- Coulomb interaction makes it hard to fill 2 e^{-}s in an orbital

$$H = -t \sum_{\langle \ell, m \rangle} c_\ell^\dagger c_m + U \sum_\ell n_{\ell\uparrow} n_{\ell\downarrow}$$: Hubbard interaction H_U

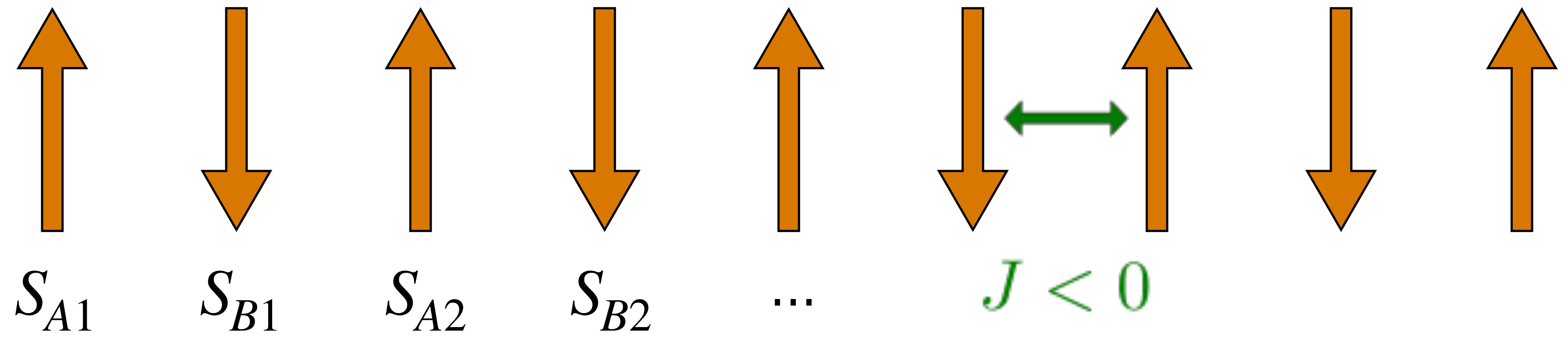
  - In the half-filled materials, large U enforces $n_{\ell\uparrow} + n_{\ell\downarrow} = 1$, making the system a Mott insulator



- In the large U limit, we obtain an effective spin-spin exchange interaction

$$H_{\rm eff} \sim H_t \frac{1}{H_U} H_t = \frac{t^2}{U} \sum_{\langle \ell, m \rangle} \vec{S}_\ell \cdot \vec{S}_m$$

  - Since $J = -t^2/U < 0$, the system acquires an anti-ferromagnetic ordering

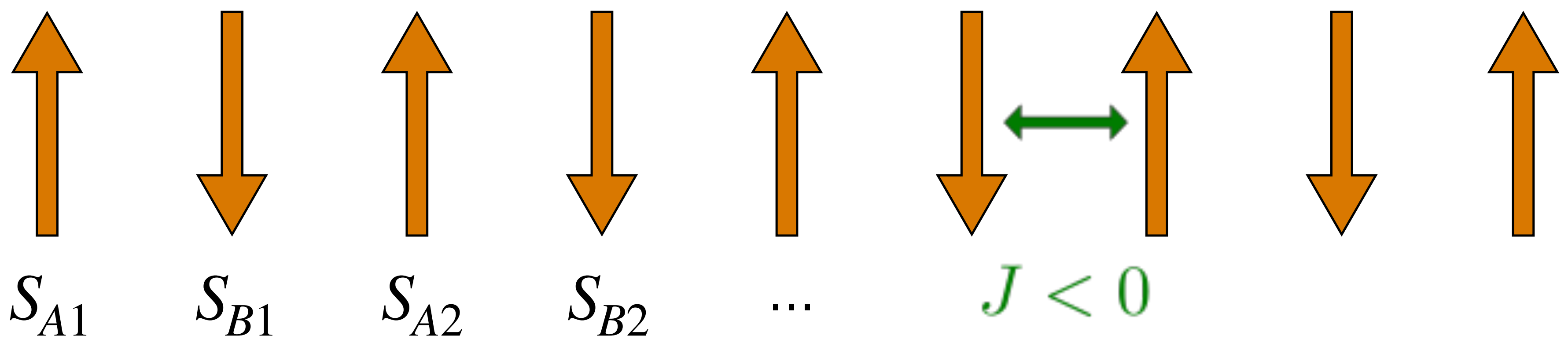


Induced spin-electron interaction

- Define an anti-ferromagnetic (Néel) order parameter

  $\left\langle \vec{S}_{\ell,A} \right\rangle = - \left\langle \vec{S}_{\ell,B} \right\rangle \equiv \vec{m}$

  for sublattices A & B



- Apply the mean-field approximation to $H_U = U \sum_\ell n_{\ell\uparrow} n_{\ell\downarrow}$

$$H_U \simeq U \sum_\ell \left(\langle n_{\ell\uparrow} \rangle n_{\ell\downarrow} + \langle n_{\ell\downarrow} \rangle n_{\ell\uparrow} - \langle n_{\ell\uparrow} \rangle \langle n_{\ell\downarrow} \rangle - \langle c^\dagger_{\ell\uparrow} c_{\ell\downarrow} \rangle c^\dagger_{\ell\downarrow} c_{\ell\uparrow} - \langle c^\dagger_{\ell\downarrow} c_{\ell\uparrow} \rangle c^\dagger_{\ell\uparrow} c_{\ell\downarrow} + \langle c^\dagger_{\ell\uparrow} c_{\ell\downarrow} \rangle \langle c^\dagger_{\ell\downarrow} c_{\ell\uparrow} \rangle \right)$$

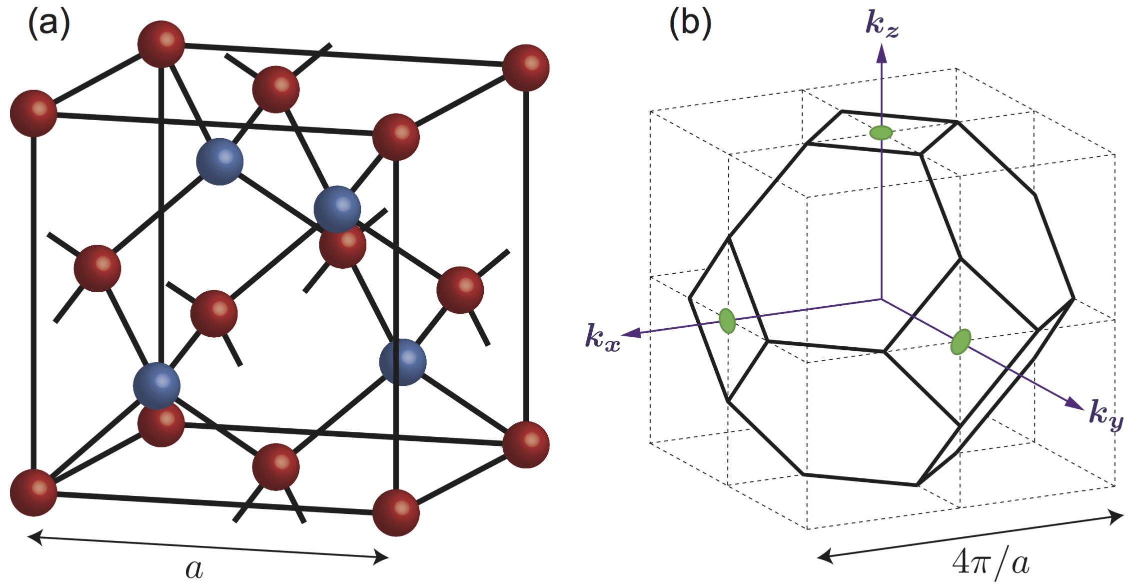
$$= U \sum_\ell \left(\frac{1}{2} + \langle S^z_\ell \rangle \right) n_{\ell\downarrow} + \left(\frac{1}{2} - \langle S^z_\ell \rangle \right) n_{\ell\uparrow} - \langle S^+_\ell \rangle c^\dagger_{\ell\downarrow} c_{\ell\uparrow} - \langle S^-_\ell \rangle c^\dagger_{\ell\uparrow} c_{\ell\downarrow} + \cdots$$

$$S^a_\ell = \frac{1}{2} \begin{pmatrix} c^\dagger_{\ell\uparrow} & c^\dagger_{\ell\downarrow} \end{pmatrix} \sigma^a \begin{pmatrix} c_{\ell\uparrow} \\ c_{\ell\downarrow} \end{pmatrix}$$

- This gives an axionic interaction btw $\vec{m}$ and Dirac e^-s in the low energy

$$S = \int d^4x \sum_{r=1,2,3} \overline{\psi}_r [i\gamma^\mu(\partial_\mu - ieA_\mu) - \delta t - i\gamma_5 U m_r] \psi_r \quad \text{with} \quad \gamma_5 = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}, \quad \psi \sim (A\uparrow, A\downarrow, B\uparrow, B\downarrow)$$

  - It is axionic because the order parameter has opposite signs for A/B e^-s
  - Three Dirac e^-s $\psi_r (r = 1,2,3)$ couple to the corresponding m_r

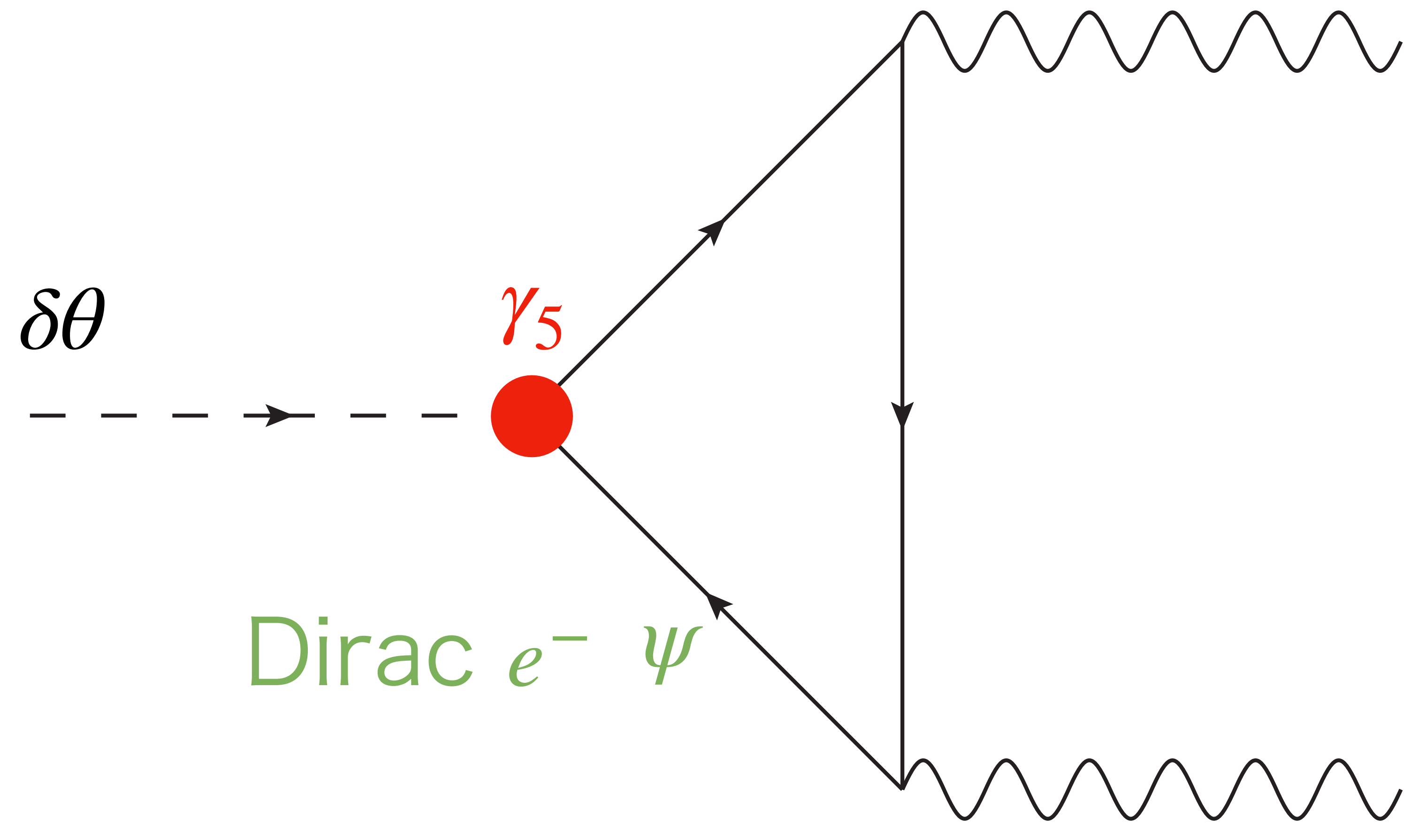


A. Sekine, K. Nomura '14

$E \ll \delta t$ phenomenology

- $E \ll \delta t$ phenomenology after integrating out Dirac e^-s

$$S_\theta = \frac{\alpha_e}{4\pi} \int d^4x\, \theta F_{\mu\nu} \widetilde{F}^{\mu\nu} \qquad \theta \equiv \pi + \sum_r \theta_r = \pi + \sum_r \tan^{-1}\left(\frac{Um_r}{\delta t}\right)$$



- Also, the dynamical fluctuation $\vec{m} \to \vec{m} + \delta\vec{m}$ gives the dynamical axion mode $\delta\theta$

$$\Delta S_\theta = \frac{\alpha_e}{4\pi} \int d^4x\, \delta\theta\, F_{\mu\nu} \widetilde{F}^{\mu\nu}$$

$$\delta\theta \simeq \frac{1}{4} \sum_r \frac{U/\delta t}{1 + U^2 m_r^2/\delta t^2} \delta m_r$$

- This can also be rewritten as a linear combination of magnons $\tilde{\alpha}_0, \tilde{\beta}_0$

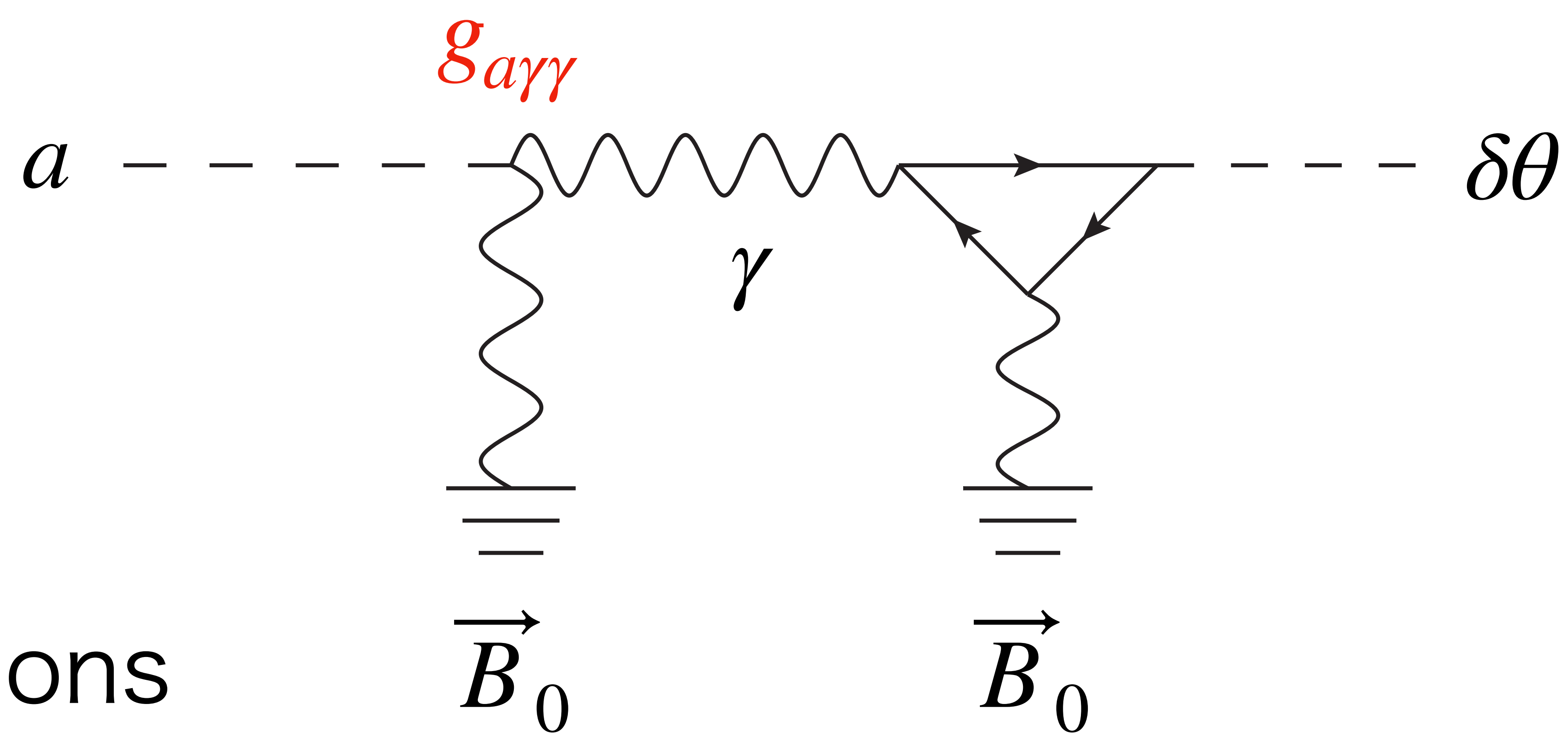
$$\delta\theta \simeq \sqrt{\frac{s}{2N}} (u_0 - v_0) \left[D^* \tilde{\alpha}_0^\dagger - D \tilde{\beta}_0^\dagger + \mathrm{h.c.} \right]$$

Axion to axion conversion

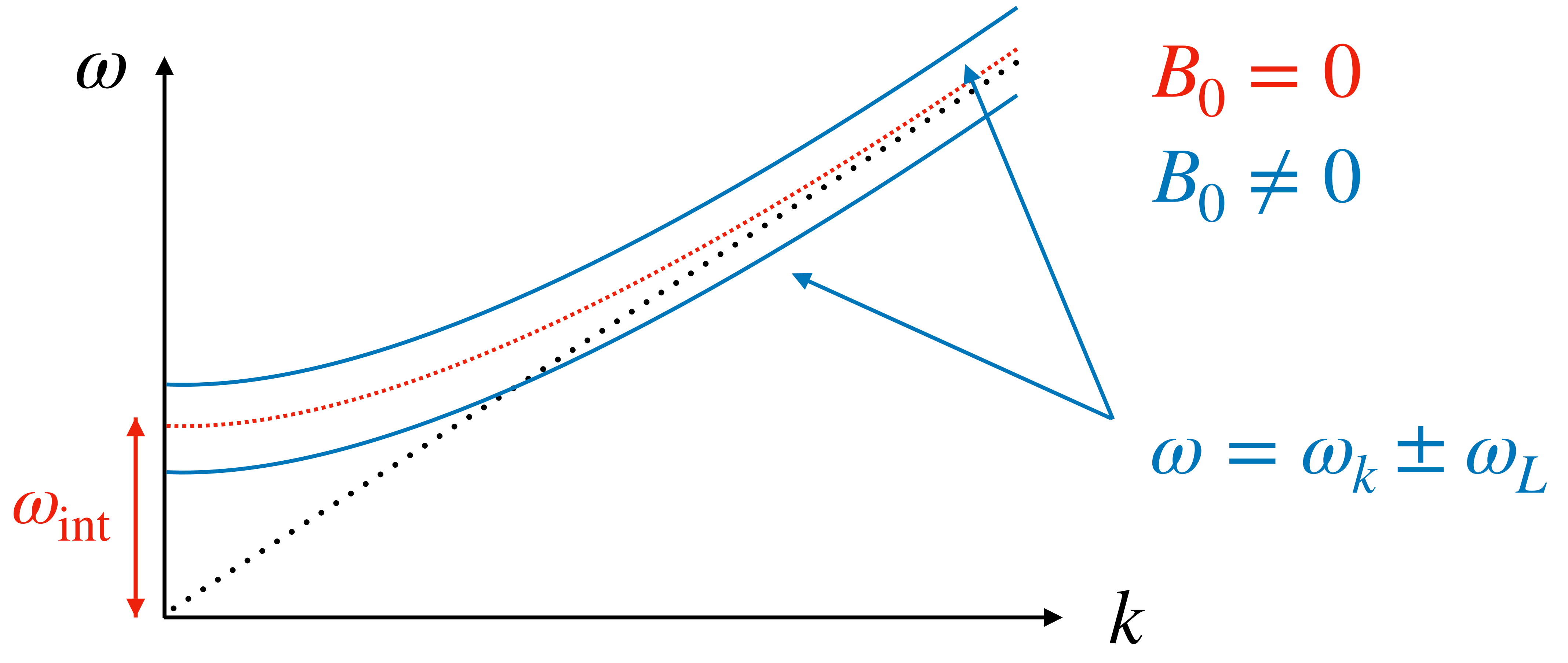
- Under background $\vec{B}_0$, axion oscillation generates an effective electric field

  $\vec{E}_a(\vec{x}, t) \simeq E_0 \hat{z} \cos(m_a t + \delta)$ with $E_0 = -\frac{1}{\epsilon} g_{a\gamma\gamma} a_0 B_0$

  - Uniform classical field same as $\vec{B}_a$



- $\vec{E}_a$ excites an axion = a linear combination of magnons
  - Resonance at $m_a \simeq \omega_{\text{int}} \pm \omega_L$
  - ω_{int} : Intrinsic gap
  - $\tilde{\alpha}_0$: AF magnon with spin ↑, $\omega = \omega_{\text{int}} + \omega_L$
  - $\tilde{\beta}_0$: AF magnon with spin ↓, $\omega = \omega_{\text{int}} - \omega_L$



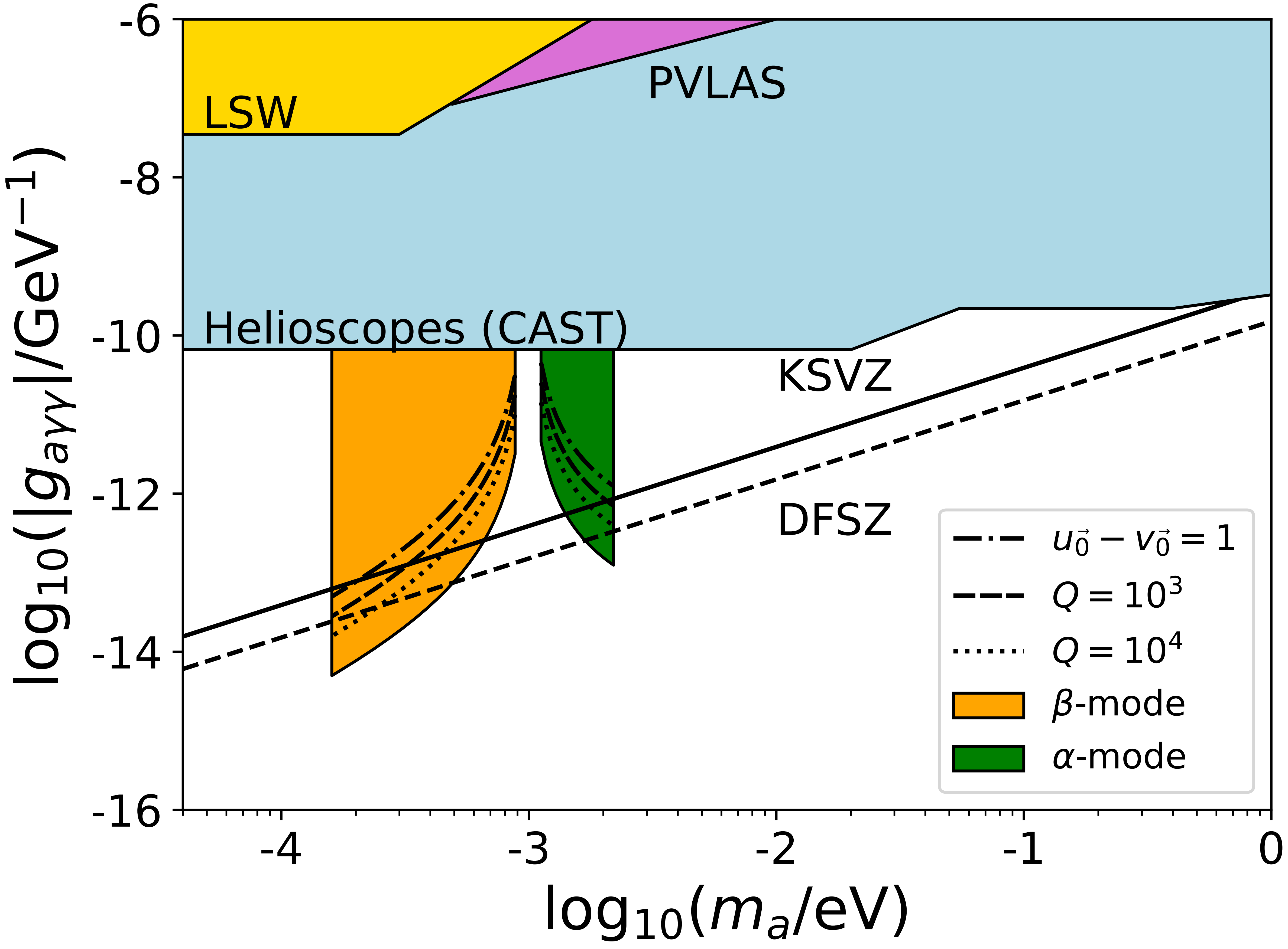
Sensitivity on axion DM

- Illustration of possible sensitivity curves

- "Plausible" choice of parameters
  - $\omega_{\rm int} = 1\,{\rm meV}$
  - $B_0 \sim \mathcal{O}(1)\,{\rm T}$
  - etc.

- A different model of axionic material as the TOORAD experiment

David J. E. Marsh+ '19, J. Schütte-Engel+ '21



S. Chigusa, T. Moroi, K. Nakayama [2102.06179]

Conclusion

- Spin dynamics in materials give us various approaches to axion DM search!

	axion	magnon	NV center	Nuclear spin
Target mass	$\sim \mathcal{O}(100)\,\mathrm{meV}$ (depends on anisotropy)	$\sim \mathcal{O}(100)\,\mathrm{meV}$ (depends on anisotropy)	$\lesssim 0.1\,\mathrm{meV}$	$1\,\mu\mathrm{eV} \sim$
Approach	Resonance scan external B_0 .	Resonance scan external B_0 .	Broadband can also focus on fixed m_a.	Resonance scan external B_0 .
DM coupling	$g_{a\gamma\gamma}$ no v_a supression	g_{aee}	g_{aee}	g_{aNN}
target materials	axionic materials ex) AF topo. insulator	magnetic material ex) YIG	(pink) diamond	Superfluid $^3\mathrm{H_e}$. $\mathrm{M_nCO_3}$

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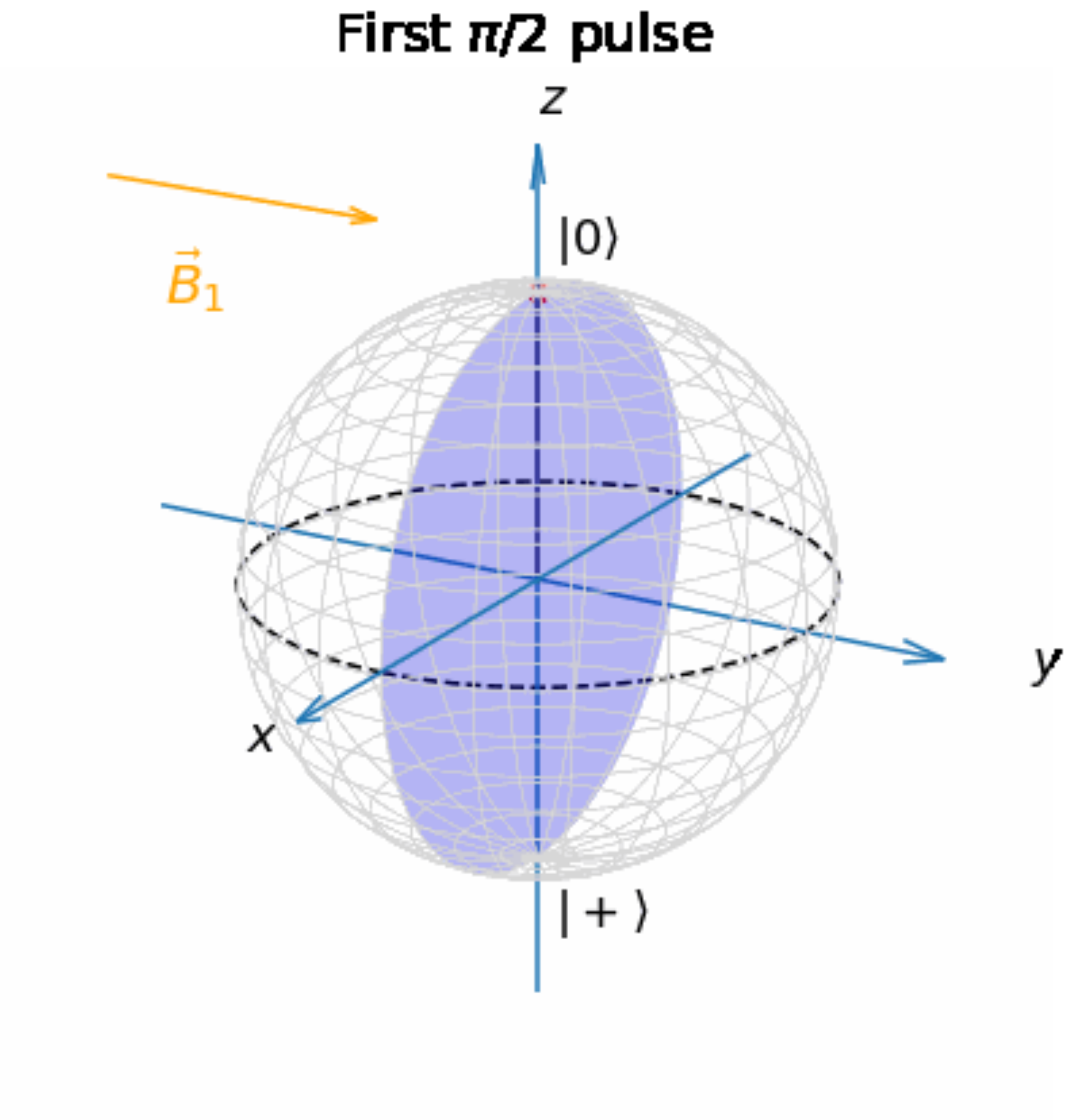
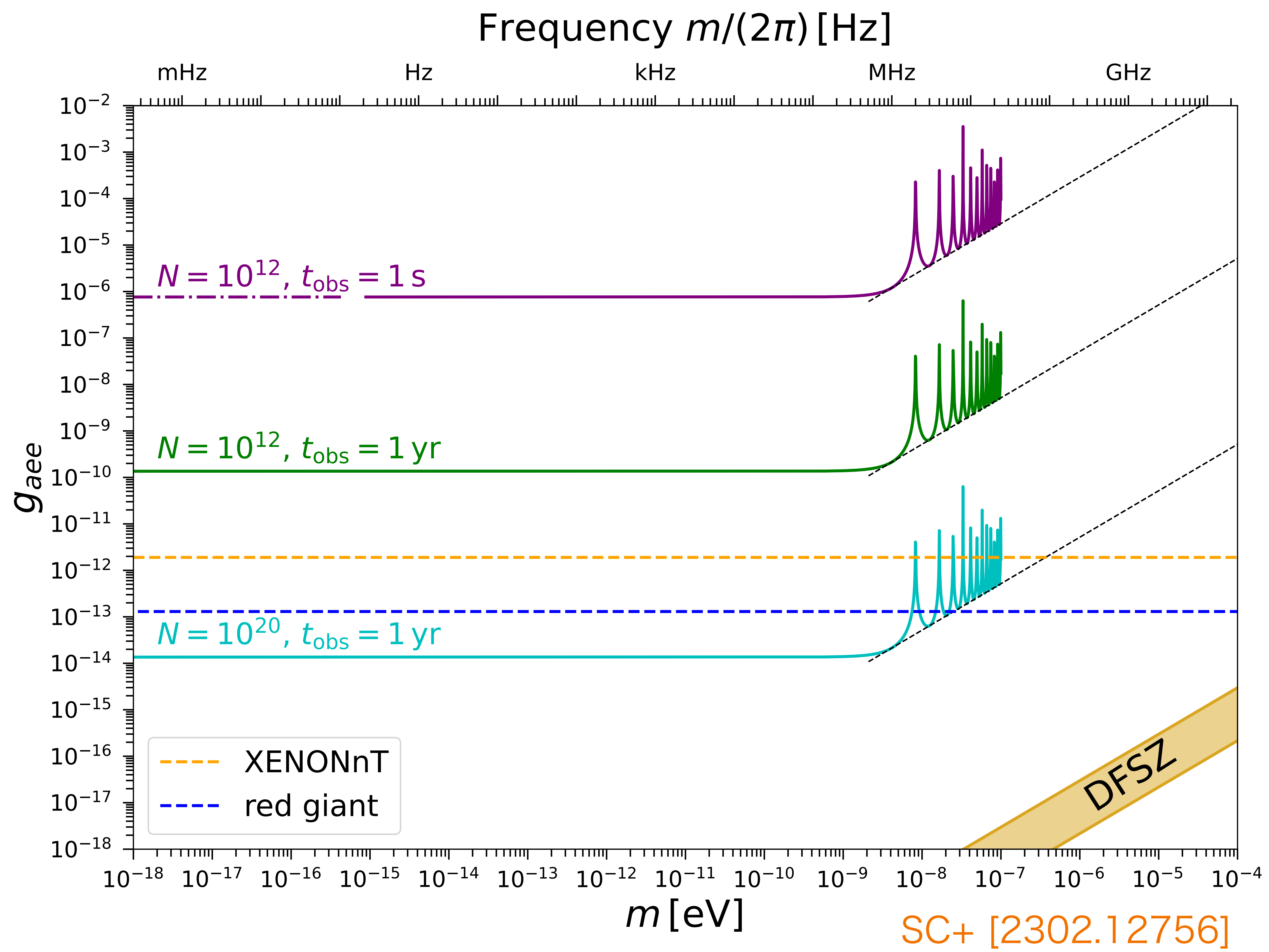
target materials	axionic materials ex) AF topo. insulator	magnetic material ex) YIG	(pink) diamond	Superfluid $^3\mathrm{H_e}$. $\mathrm{M_nCO_3}$

Thank you!

Backup slides

The NV center sensitivity on axion DM





SC+ [2302.12756]

- Sensitivity on g_{aee} for a broad mass range $m_a \lesssim 10^{-4}\,\mathrm{eV}$

Relationship w/ topology

✓ Normal / Topological insulators have different topologies = No continuous deformation

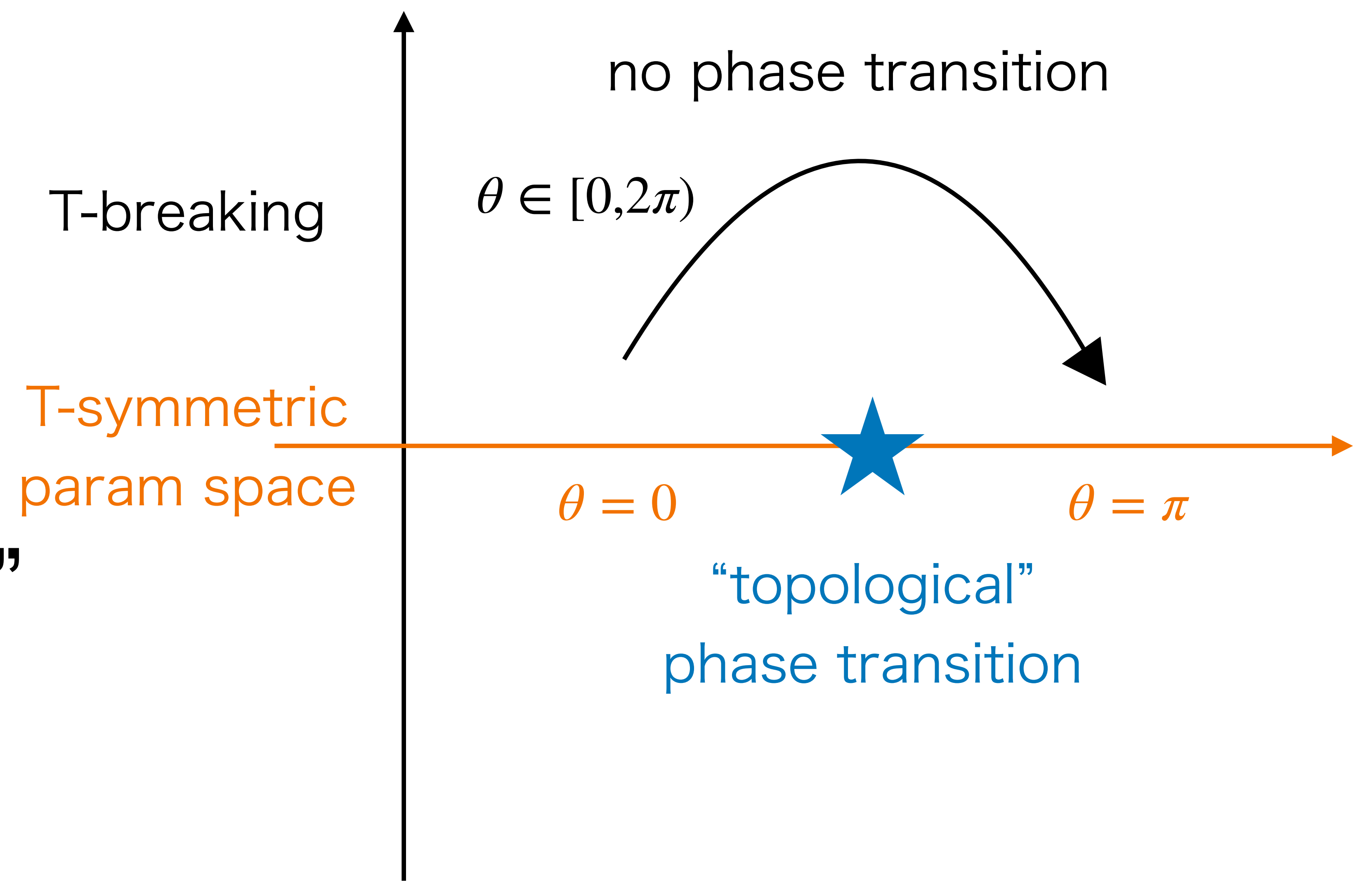
- Topological invariant θ is evaluated w/ berry connection

$$\mathscr{A}_i^{\alpha\beta} = -i\langle u_k^\alpha | \frac{\partial}{\partial k_i} | u_k^\beta \rangle \qquad \theta \equiv \frac{1}{4\pi}\int_{\mathrm{BZ}} d^3k\, \epsilon^{ijk}\, \mathrm{Tr}\left[\mathscr{A}_i \partial_j \mathscr{A}_k + i\frac{2}{3}\mathscr{A}_i \mathscr{A}_j \mathscr{A}_k\right]$$

Berry connection   Bloch states↔energy eigenstates   Brillouin zone

- Time reversal symmetry forces θ to be one of below
  - $\theta = 0$ (normal insulator)
  - $\theta = \pi$ (topological insulator)
- SPT phase

  = "symmetry protected topological phase"



θ is axion term

✓ Topological EM response

$$S = \frac{\alpha}{4\pi}\int dt d^3x\, \theta\, \underbrace{F_{\mu\nu}\widetilde{F}^{\mu\nu}}_{=4\vec{E}\cdot\vec{B}} \quad ; \quad \widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

- θ is "static" axion term
- $\vec{B}$ induces electric polarization $\vec{P} \propto \theta\vec{B}$
- $\vec{E}$ induces magnetization $\vec{M} \propto \theta\vec{E}$

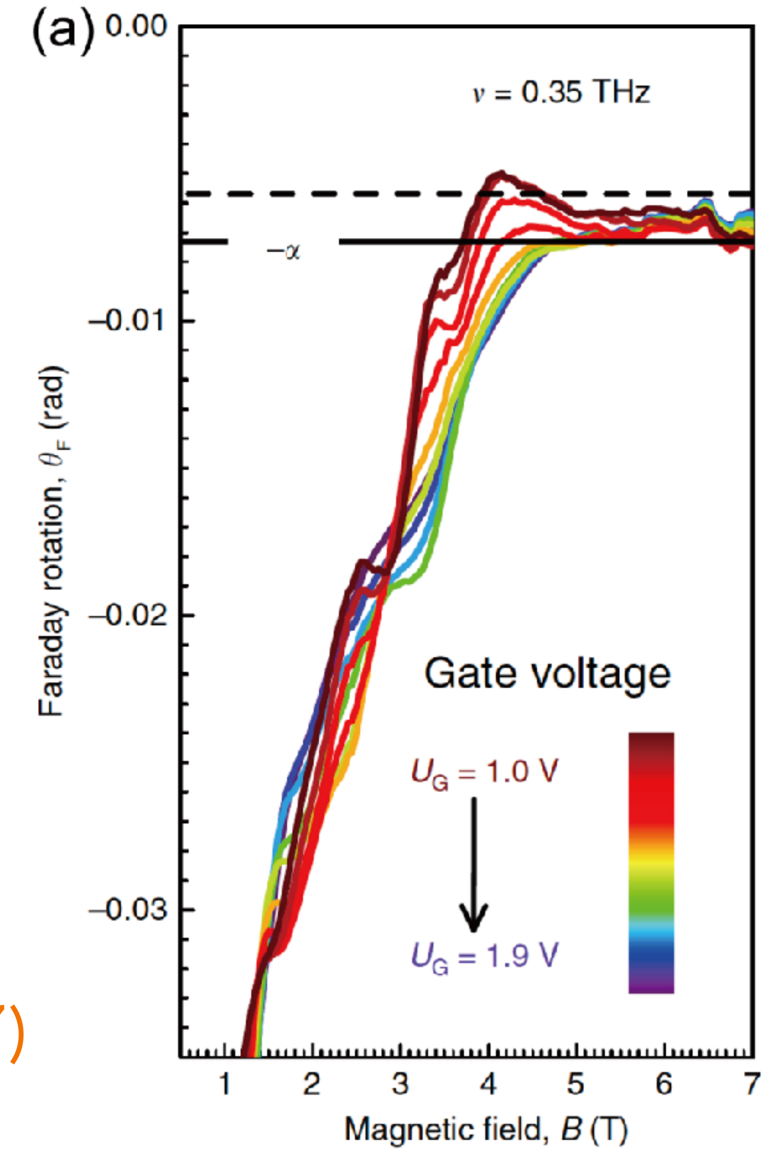
✓ Rich phenomenology like

- Faraday rotation

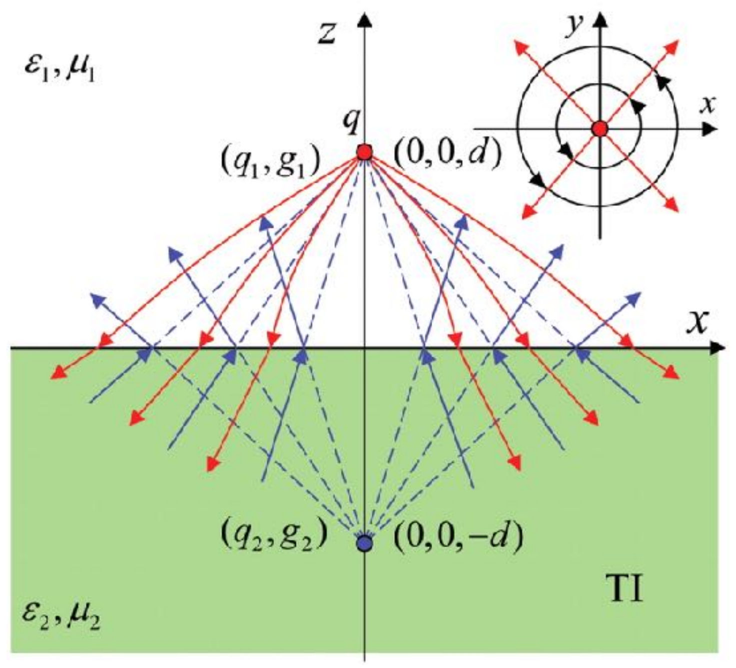
  rotation of polarization plane of linearly polarized photon

  cf. cosmological birefringence

  V. Dziom+ Nat. Commun. 8, 15197 (2017)



- Image monopole effect



X. Qi+ Science 323, 1184 (2009)

Emergence of magnetic fields as if "image monopole" exists

Order estimate of physical parameters

$$\mathscr{L}_{\rm int} = \frac{\alpha}{2\pi}\frac{1}{f_{\rm CM}}\delta\theta\, F\tilde{F}$$

$$f_{\rm CM} \sim \left((u_0 - v_0)\,|D|\,\sqrt{\omega_0 V_{\rm unit}}\right)^{-1}$$

$$\sim 200\,{\rm keV}\left(\frac{1}{u_0 - v_0}\right)\left(\frac{1}{|D|}\right)\left(\frac{1\,{\rm meV}}{\omega_0}\right)^{1/2}\left(\frac{(5\,\text{Å})^3}{V_{\rm unit}}\right)^{1/2}$$

$$D = \sum_r \frac{U/\delta t}{1 + U^2 m_r^2 \delta t^2}(O_{r1} - iO_{r2}) \sim \mathcal{O}(1)$$

when $U \sim \delta t$