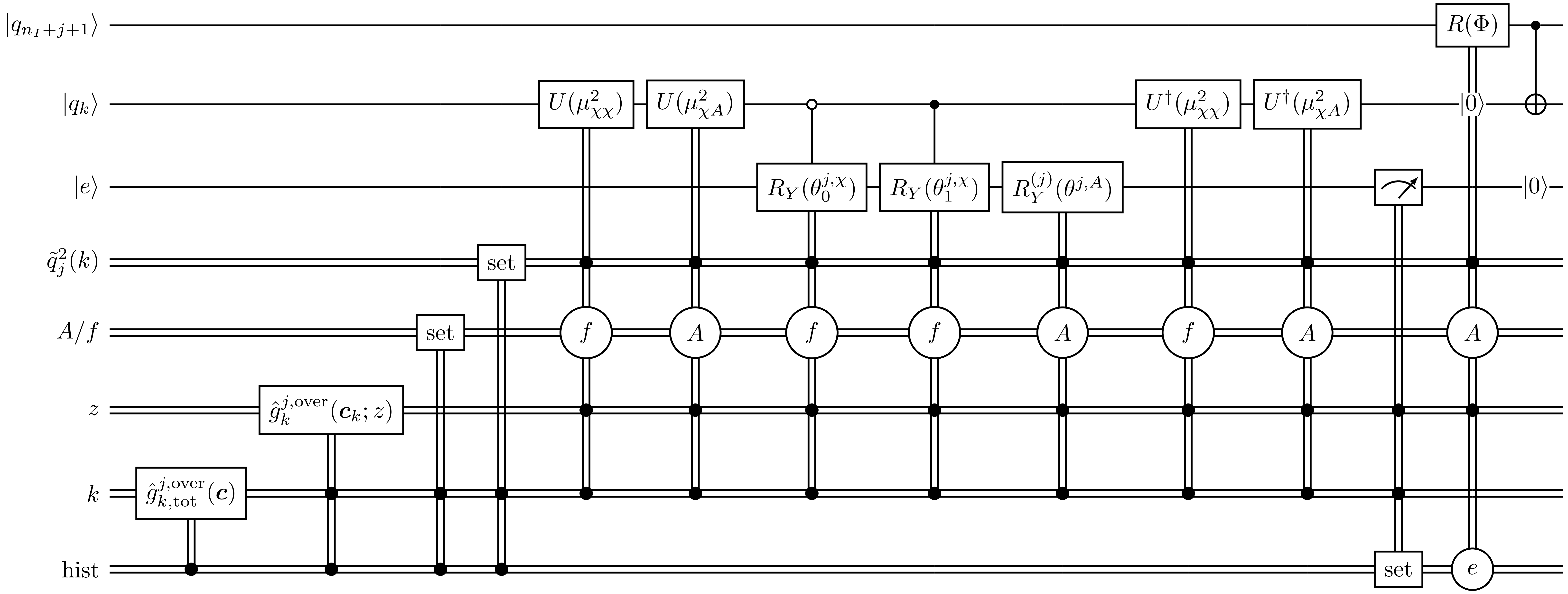
Quantum simulation of parton shower with kinematics



So Chigusa (LBNL/UC Berkeley)
with C. W. Bauer and M. Yamazaki
PLB 834 (2022) 137466 [arXiv: 2204.12500]
PRA 109 (2024) 3, 032432 [arXiv: 2310.19881]

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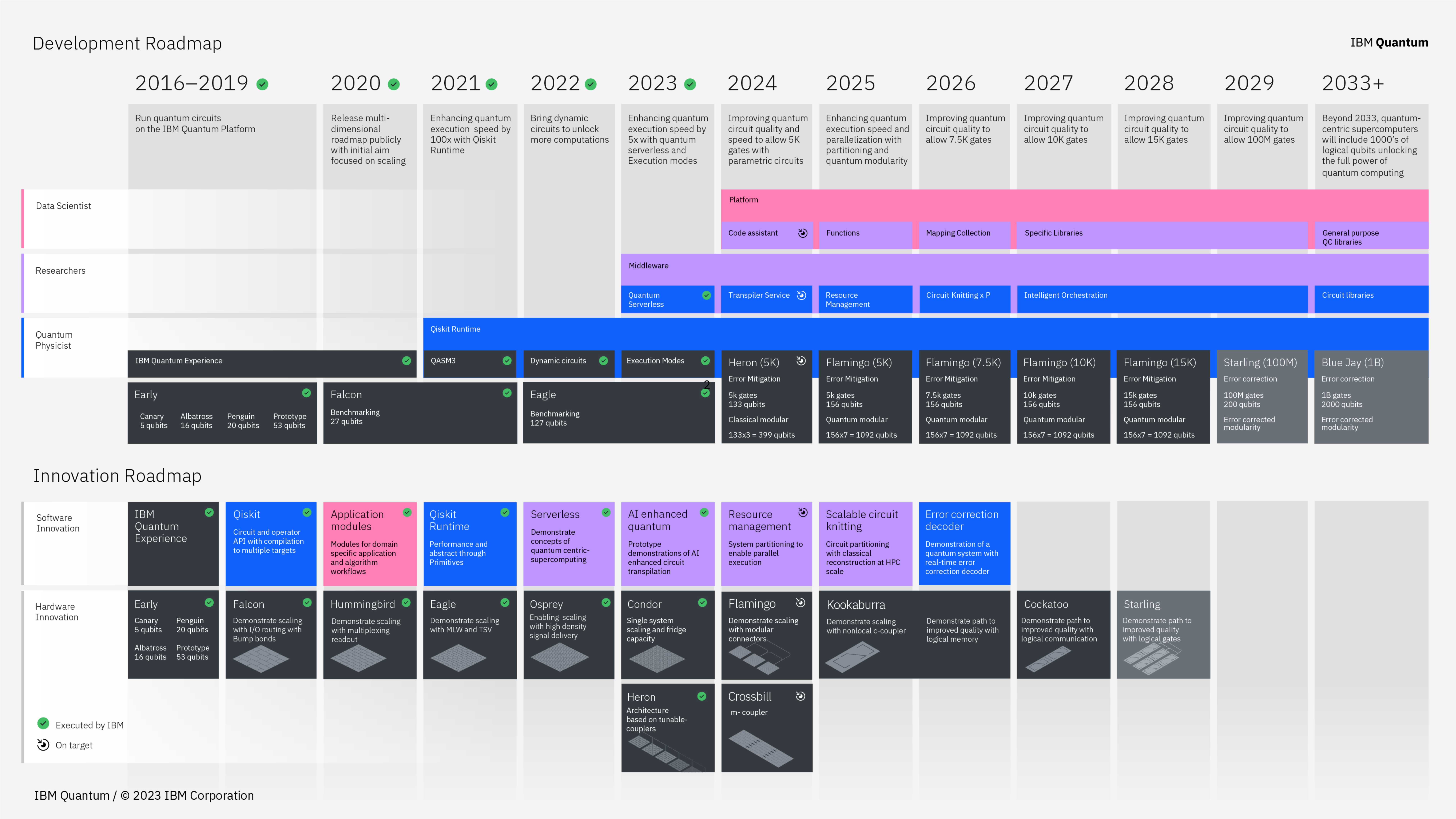
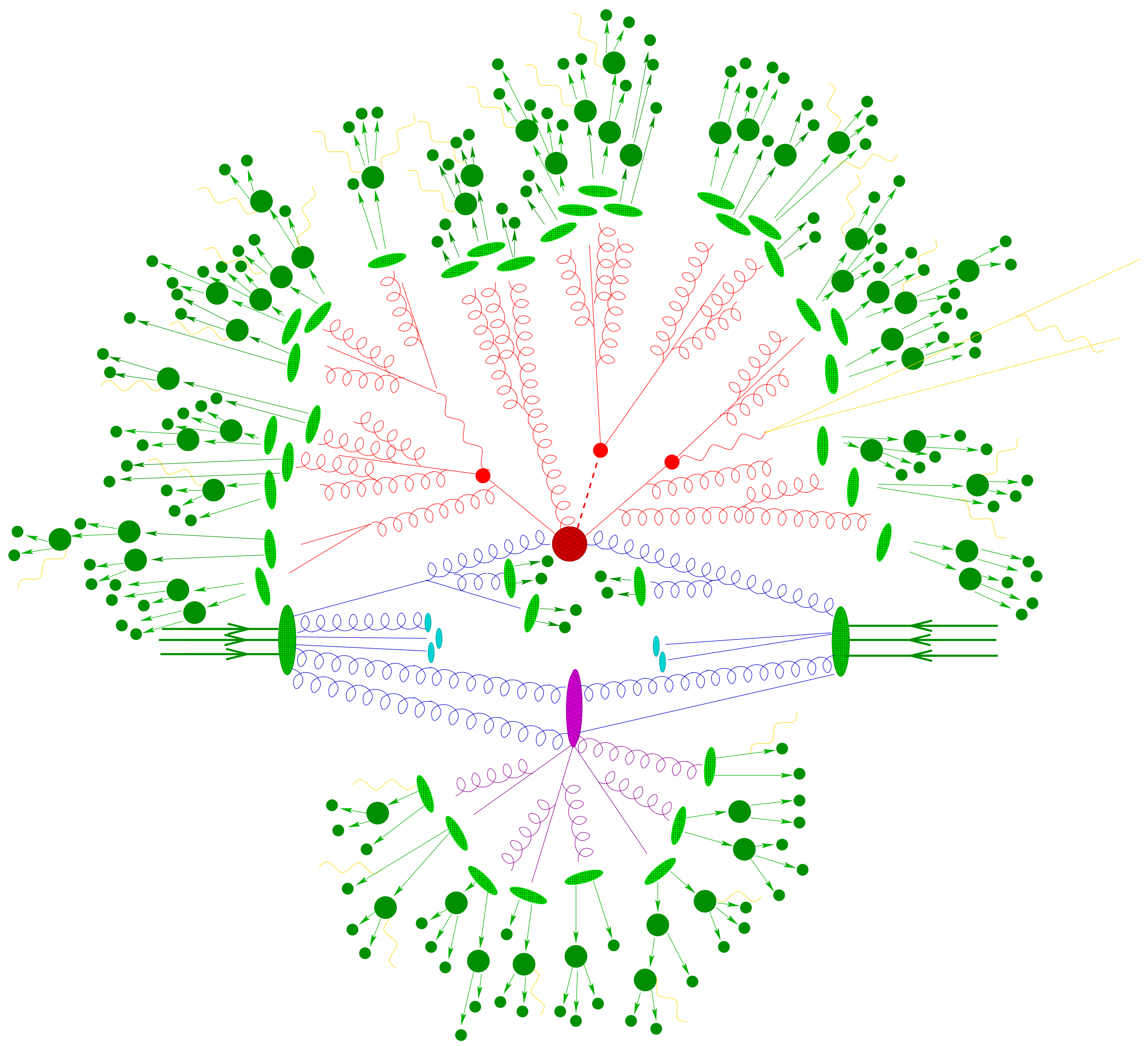
Quantum computation: NISQ era



Overview

The fact

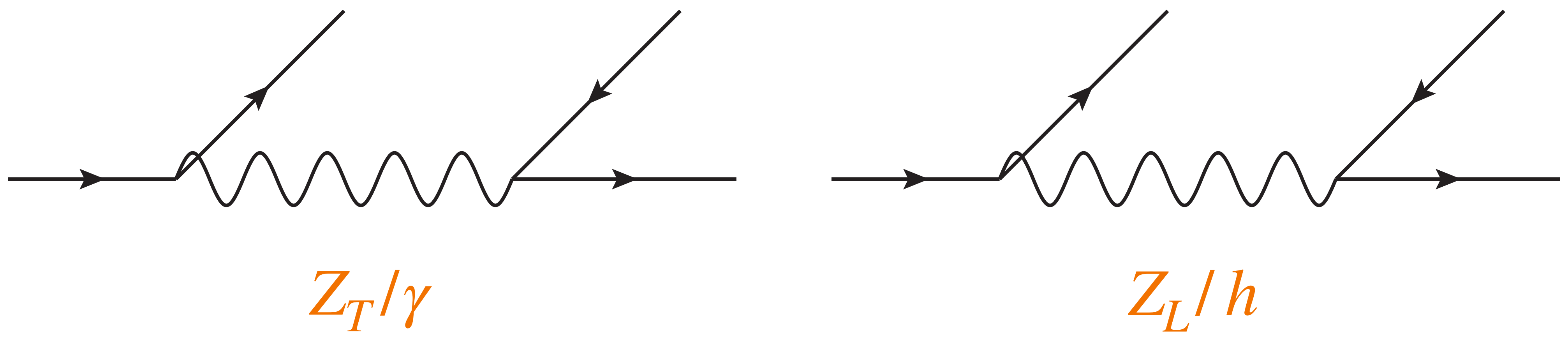
- Parton shower is a traditional algorithm to simulate high-energy multi-emission processes based on a classical probability distribution



Höche “Introduction to parton-shower event generators”

Problem

- A non-trivial “flavor” structure could induce quantum interference effects, which cannot be tracked by the classical parton shower algorithm



What we did

1. Constructed a quantum algorithm to simulate multi-emission processes, taking into account quantum interference and kinematical effects
2. Demonstrated the phenomenological implications based on a toy model

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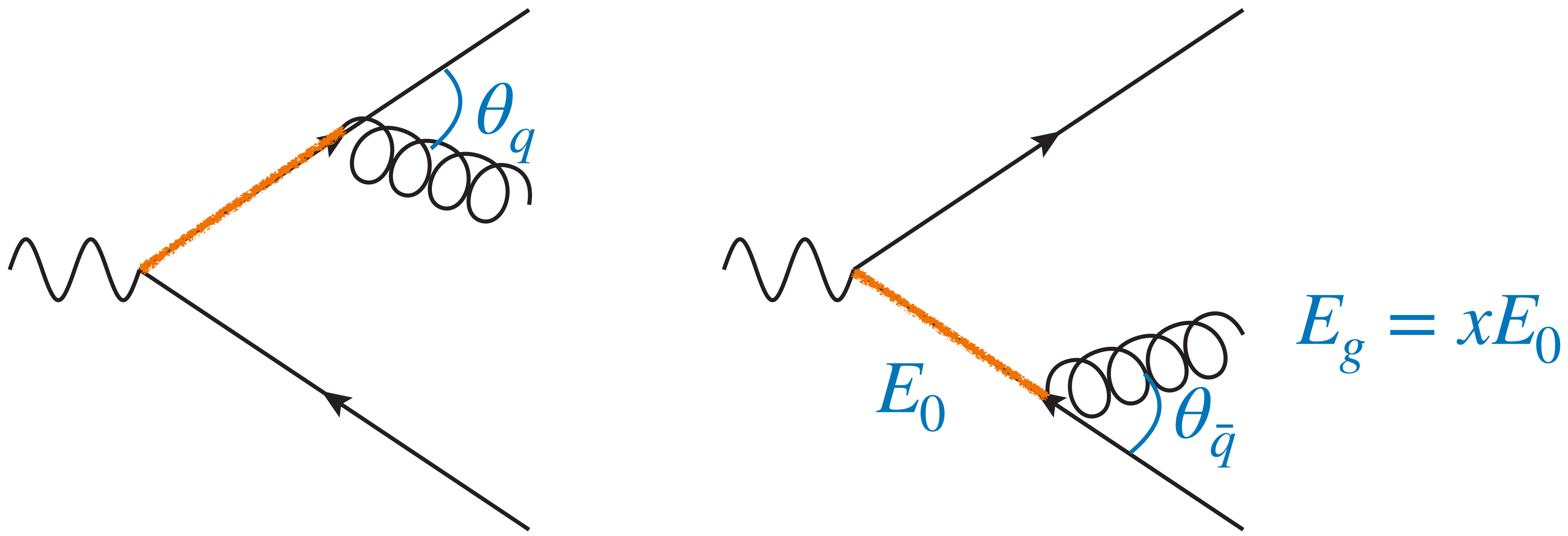
Large logarithms

- Soft/collinear singularities lead to an enhancement of emission processes
  - Ex) $q\bar{q} + g$ production



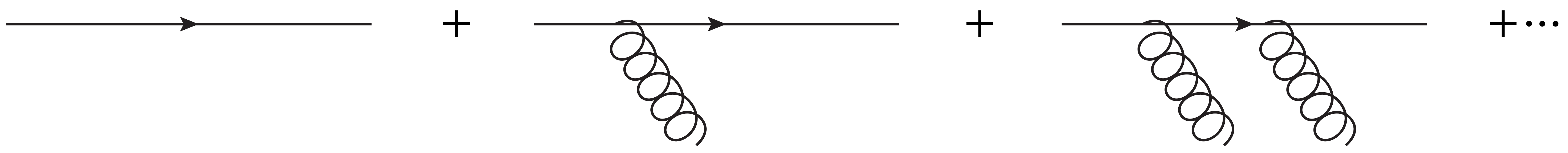
$$\frac{d\sigma_{q\bar{q}g}}{d\cos\theta\, dx} \simeq \sigma_{q\bar{q}} \sum_{f=q,\bar{q}} C_F \frac{\alpha_s}{2\pi} \frac{2}{\sin^2\theta_f} \frac{1+(1-x)^2}{x} \quad \text{and} \quad \sigma_{q\bar{q}g} \propto \sigma_{q\bar{q}} \frac{\alpha_s}{2\pi} \ln^2\left(\frac{E_0^2}{\mu_{\rm IR}^2}\right)$$

- The expansion parameter becomes larger - $\alpha \to \alpha \ln$ or $\alpha \ln^2$

Resummation of large logarithms

- Emissions are not necessarily suppressions at high energy scales
  - Collinear emission @ LHC: $\frac{\alpha_s(M_Z)}{2\pi} \ln\left(\frac{E_0^2}{\Lambda_{QCD}^2}\right) \sim 30\,\% \quad \Leftrightarrow \quad E_0 \sim 0.6\,\mathrm{TeV}$
  - Soft & collinear γ @ muon collider: $\frac{\alpha}{2\pi} \ln^2\left(\frac{E_0^2}{m_\mu^2}\right) \sim 30\,\% \quad \Leftrightarrow \quad E_0 \sim 1\,\mathrm{TeV}$
  - Collinear emission from heavy DM: $\frac{\alpha_2(M_Z)}{2\pi} \ln\left(\frac{E_0^2}{m_Z^2}\right) \sim 30\,\% \quad \Leftrightarrow \quad E_0 \sim 0.5\,\mathrm{EeV}$

C. W. Bauer, et al. [2007.15001]



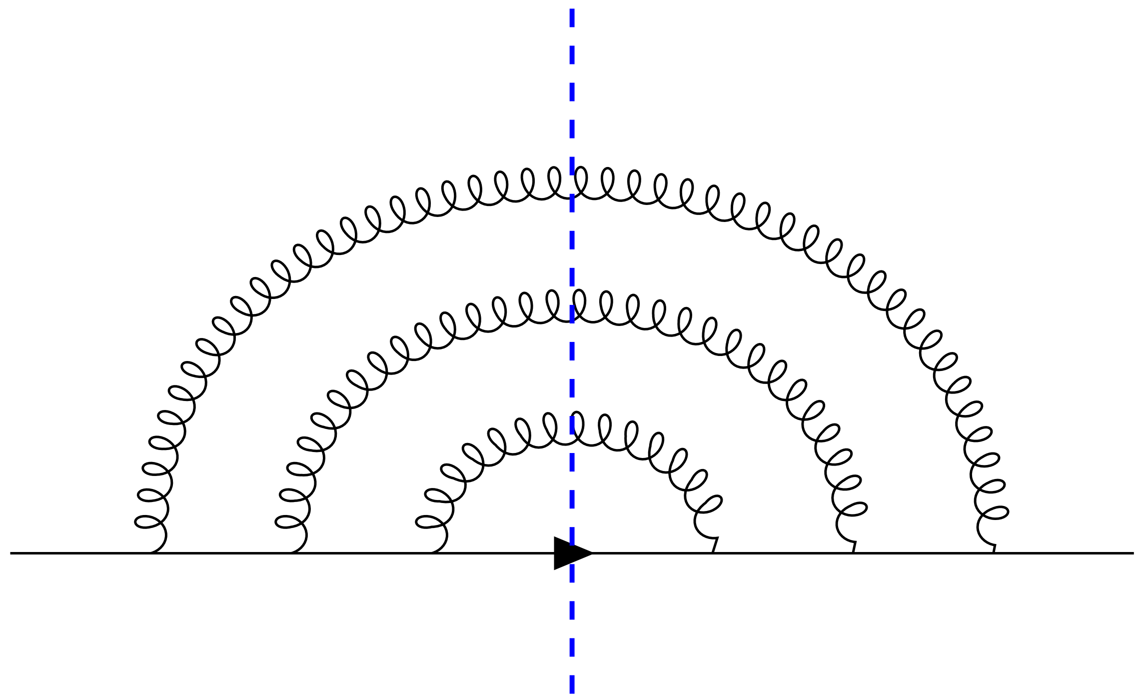
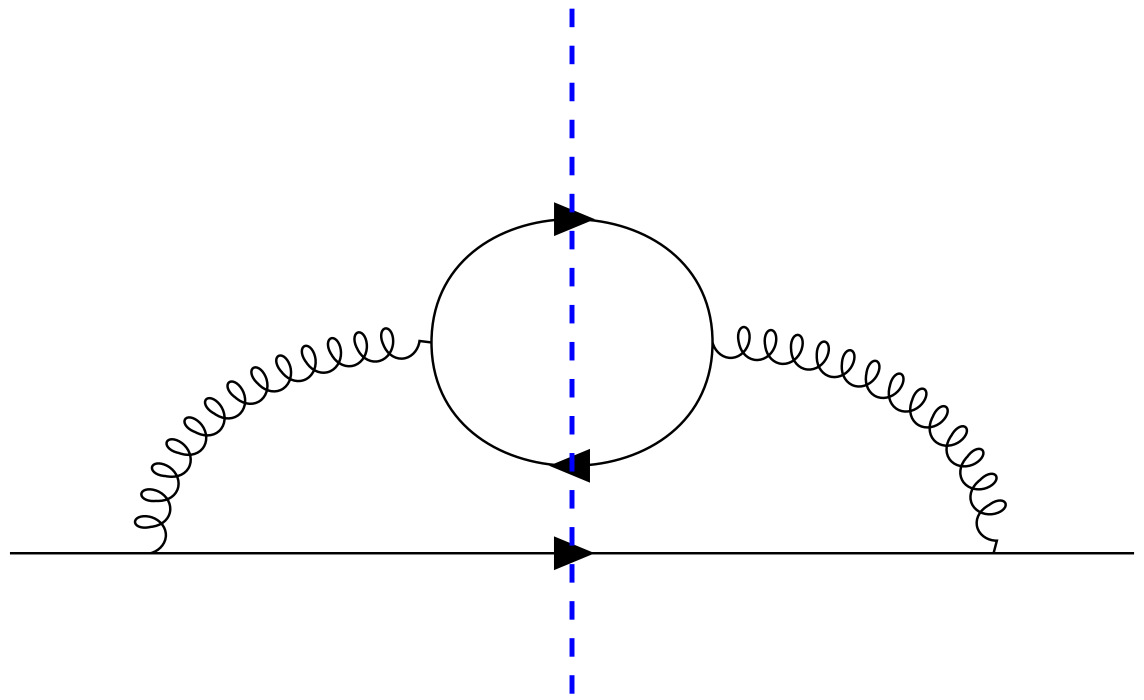
- Resummation of large logs needed! * The leading logarithms (LL), $\sum_n (\alpha \ln)^n$

Coherence

Only ladder-type diagrams with 1 → 2 splittings contribute at the LL

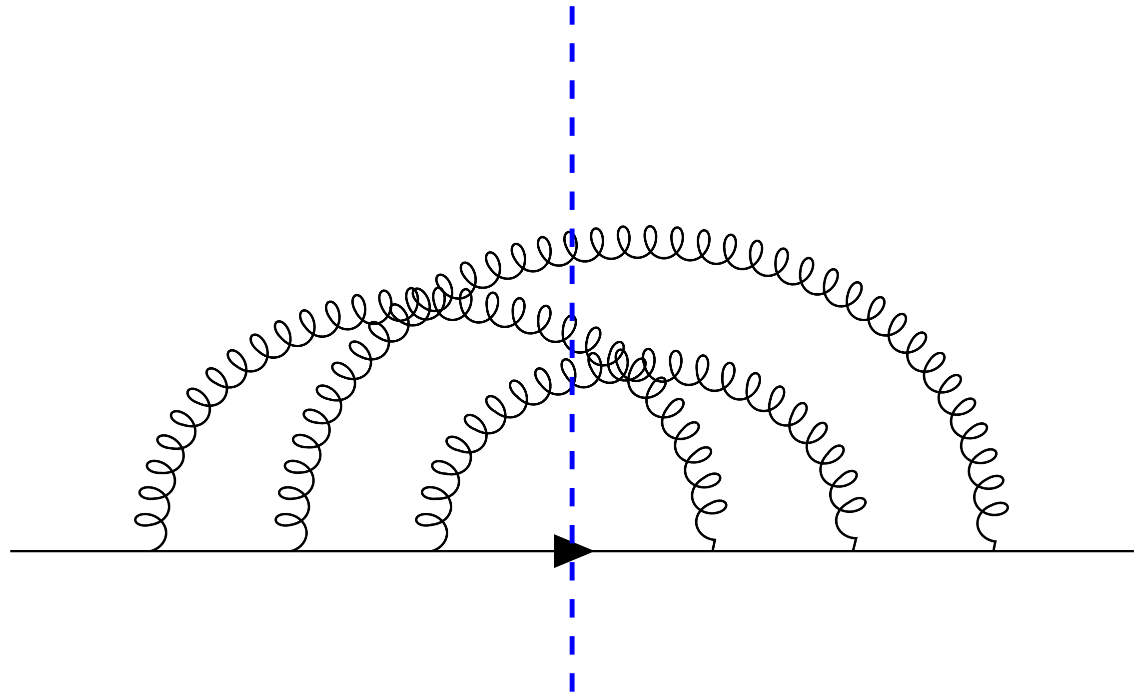
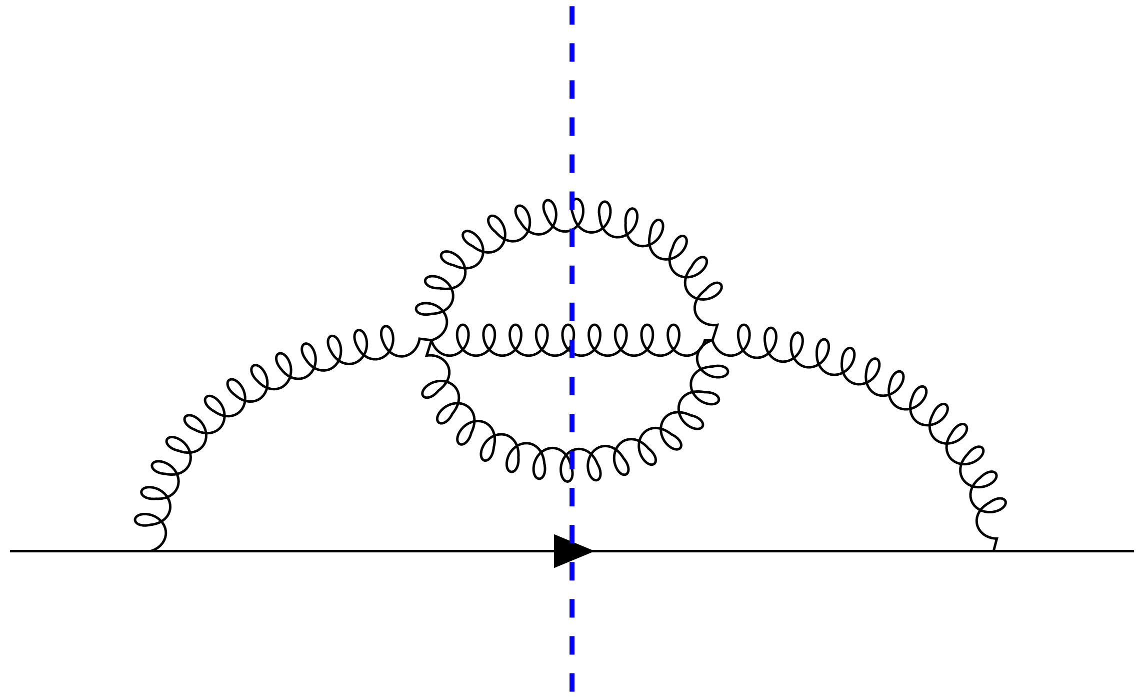
- LL contributions

$$\sum_n (\alpha \ln)^n + \cdots$$

- Beyond LL

$$\sum_n \alpha^n \ln^{n-1} + \cdots$$

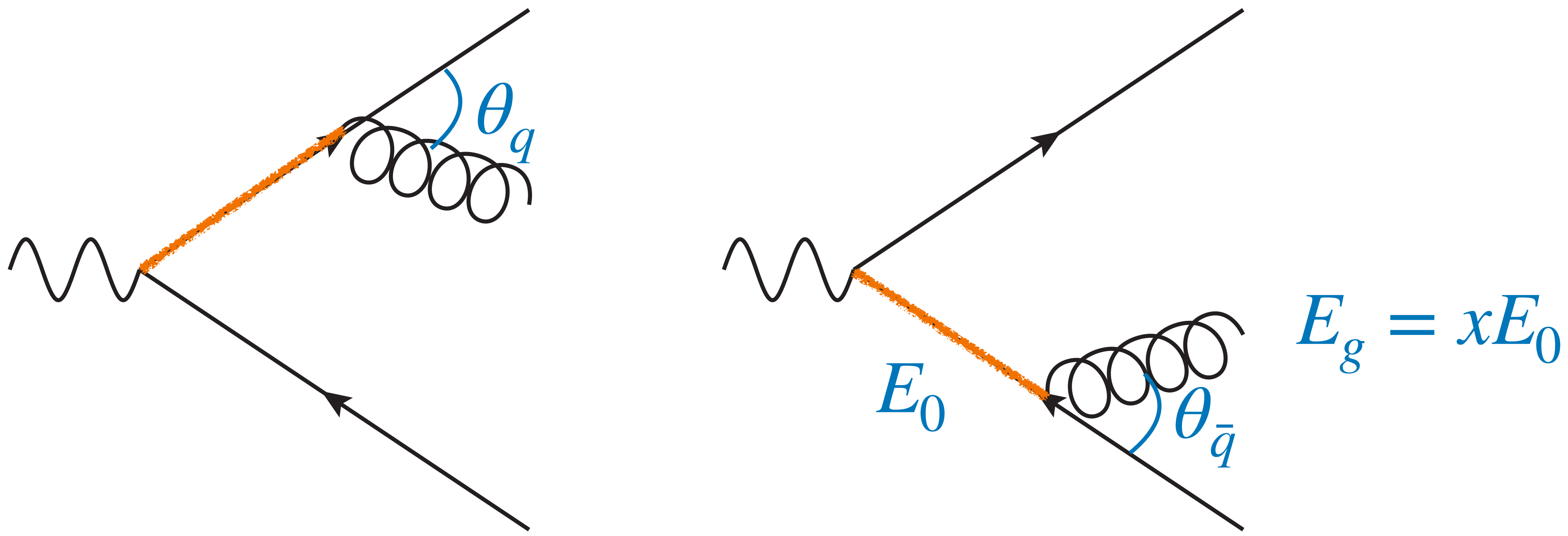



- cf) Virtuality ordering Chang+ '70, Gribov+ '72, Dokshitzer '77

cf) Angular ordering Marchesini+ '84, '88

Classical probabilistic interpretation

- The relationship among cross sections
  - Ex) $q\bar{q}+g$ production



$$\frac{d\sigma_{q\bar{q}g}}{dt\,dx} \simeq \sigma_{q\bar{q}} \sum_{f=q,\bar{q}} \frac{\alpha_s}{2\pi}\frac{1}{t_q} C_F \frac{1+(1-x)^2}{x} \quad \text{with} \quad \text{virtuality } t_q \propto \sin^2\frac{\theta_q}{2}$$

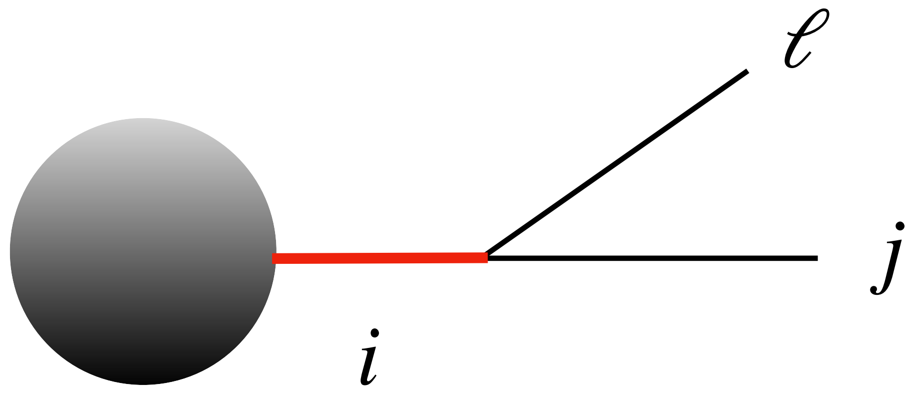
- Can be interpreted as classical "splitting probabilities"

$$d\mathcal{P}_{q\to gq} = d\mathcal{P}_{\bar{q}\to g\bar{q}} \simeq \frac{\alpha_s}{2\pi}\frac{dt}{t} C_F \frac{1+(1-x)^2}{x} dx$$

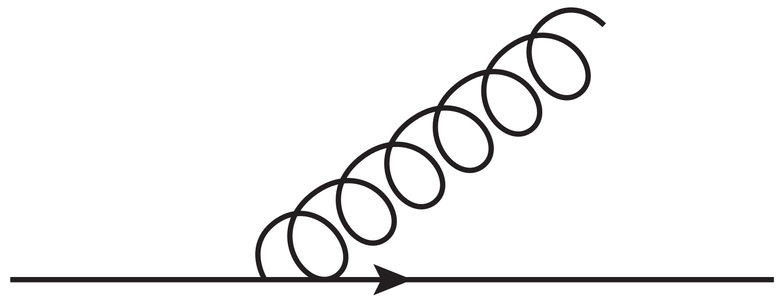
General splitting and splitting functions

- General formula of the splitting probability

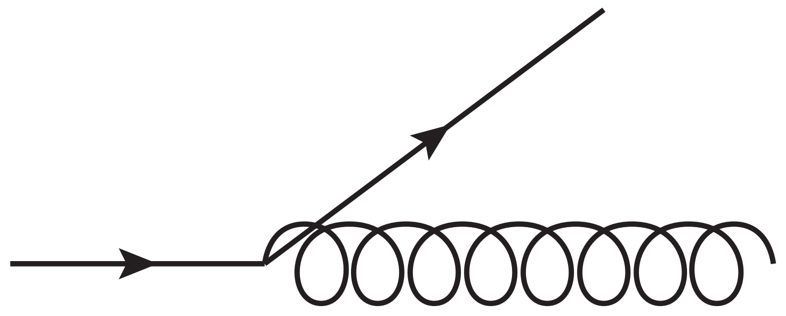
$$d\mathscr{P}_{i\to j\ell} \simeq \frac{\alpha(t,x)}{2\pi}\frac{dt}{t}P_{i\to j\ell}(x)dx$$



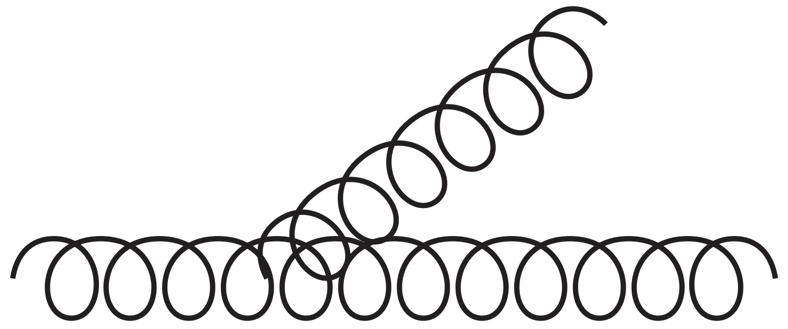
- Splitting functions in QCD



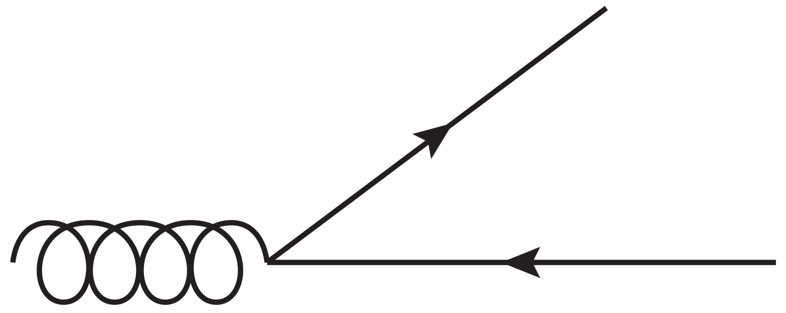
$$P_{q\to qg} = C_F\frac{1+x^2}{1-x}$$



$$P_{q\to gq} = C_F\frac{1+(1-x)^2}{x}$$



$$P_{g\to gg} = 2C_A\frac{(1-x(1-x))^2}{x(1-x)}$$



$$P_{g\to q\bar{q}} = T_R x^2(1-x)^2$$

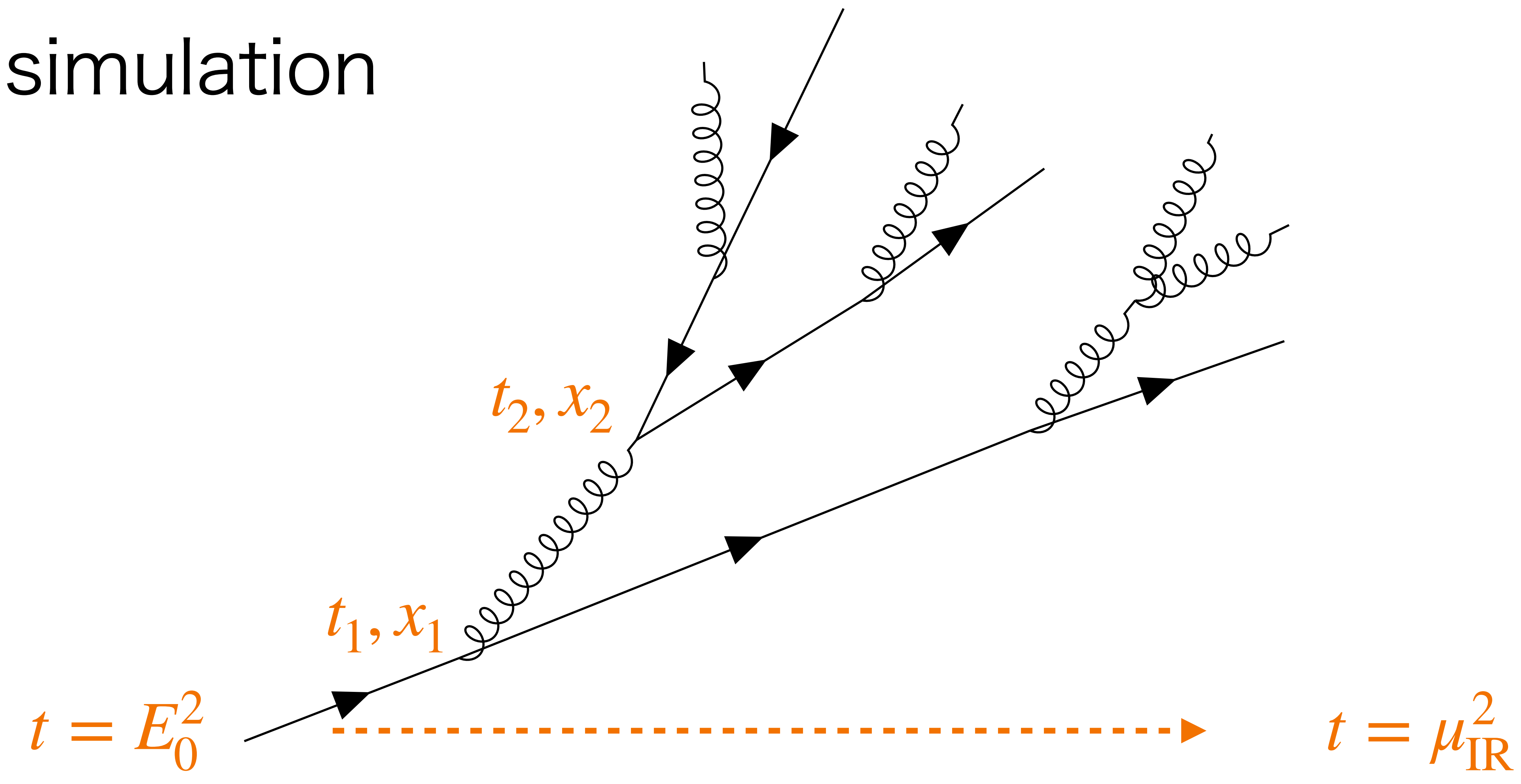
- cf) helicity effects in EW theory Chen+ [1611.00788] cf) mass effects in dark $U(1)$ Chen+ [1807.00530]

Classical parton shower

- Monte Carlo simulation to determine the multi-emission cross sections based on

$$d\mathscr{P}_{i\to j\ell} \simeq R_{i\to j\ell}(t)\,dt \quad \text{and} \quad R_{i\to j\ell}(t) \equiv \frac{1}{t}\int_{x_{\mathrm{min}}(t)}^{x_{\mathrm{max}}(t)} dx\, \frac{\alpha(t,x)}{2\pi} P_{i\to j\ell}(x)$$

- Inevitable for high-E simulation
  - Pythia8
  - Herwig
  - Sherpa
  - etc⋯



- Unitarity ensures that the inclusive cross section is unchanged
  - e.g., $\sigma_{q\bar{q}}^{\mathrm{LO}} = \sigma_{q\bar{q}}^{\mathrm{LO+LL}} + \sigma_{q\bar{q}g}^{\mathrm{LO+LL}} + \sigma_{q\bar{q}gg}^{\mathrm{LO+LL}} + \sigma_{q\bar{q}q\bar{q}}^{\mathrm{LO+LL}} + \cdots$

Analytic results for soft emissions

- Consider soft & collinear gluon emissions from a high-energy quark

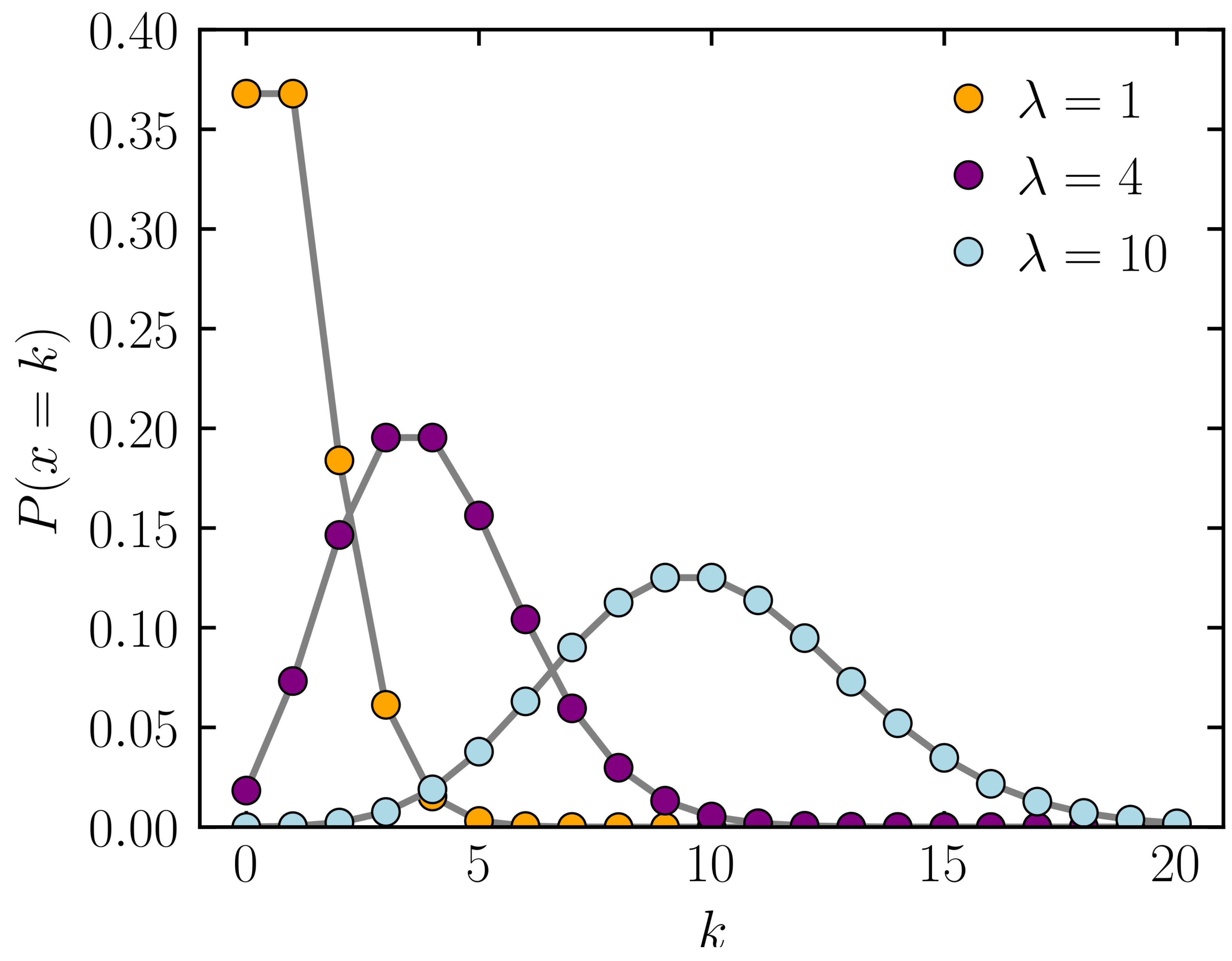
No emission probability for a given range $\Delta(t_0, t)$

$$\frac{d}{dt}\Delta(t_0,t) = -R(t)\Delta(t_0,t) \Rightarrow \Delta(t_0,t_1) = \exp\left(-\int_{t_0}^{t_1} dt\, R(t)\right)$$

$$p_0 = \Delta(\mu_{\rm IR}^2, E_0^2) \equiv e^{-\lambda} \quad \text{with} \quad \lambda \simeq C_F \frac{\alpha_s}{2\pi} \ln^2 \frac{E_0^2}{\mu_{\rm IR}^2}$$

$$p_1 \simeq \int_{t_{\rm min}}^{t_{\rm max}} dt\, \Delta(t_{\rm min}, t) R(t) \Delta(t, t_{\rm max}) = \lambda e^{-\lambda}$$

$$p_n \simeq \frac{1}{n!}\lambda^n e^{-\lambda}$$ …… probability of n gluons

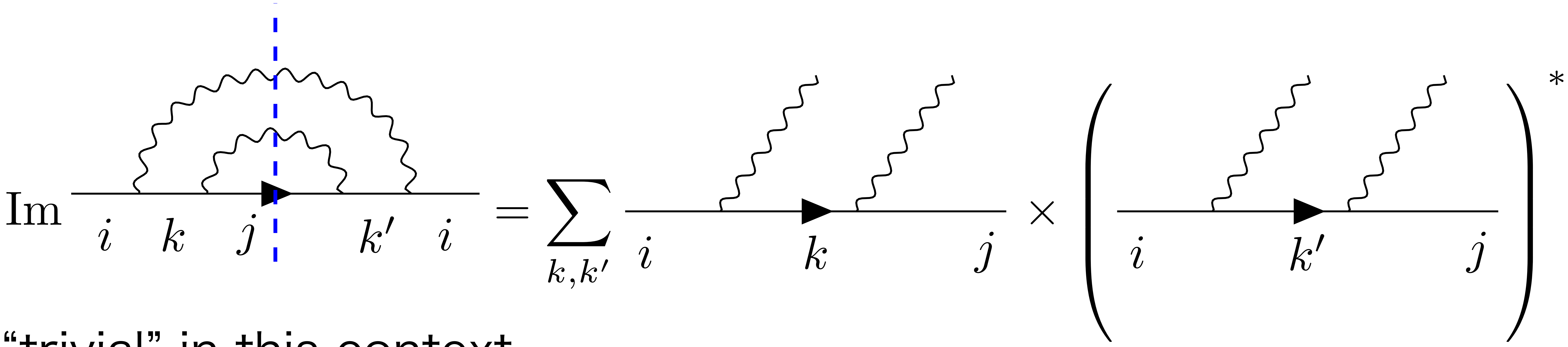


- A Poisson distribution with an average $\lambda \propto \alpha \ln^2$

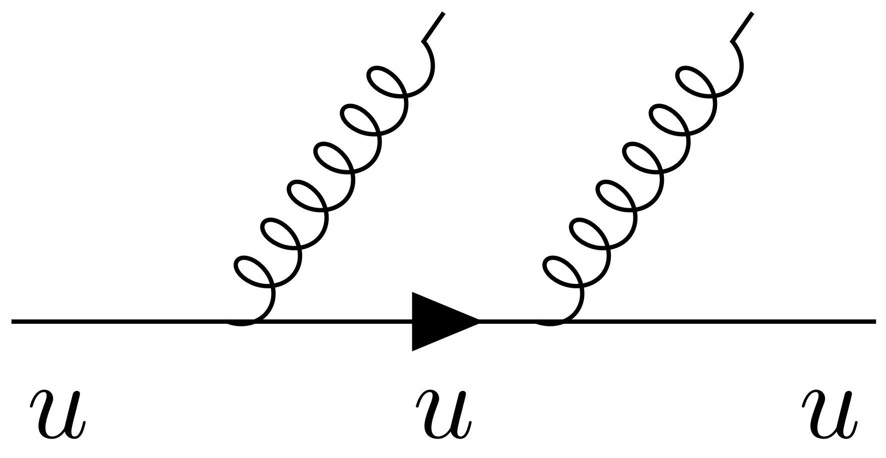
Quantum interference in parton shower

- A loophole in the discussion so far

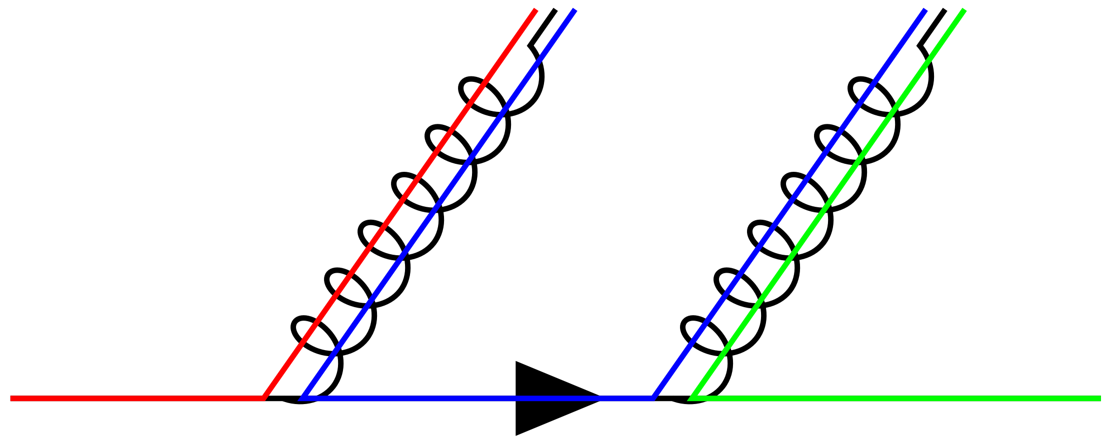
A non-trivial flavor structure makes interference effects important at the LL-level



- QCD is “trivial” in this context
  - Flavor diagonal



  - Color information is preserved

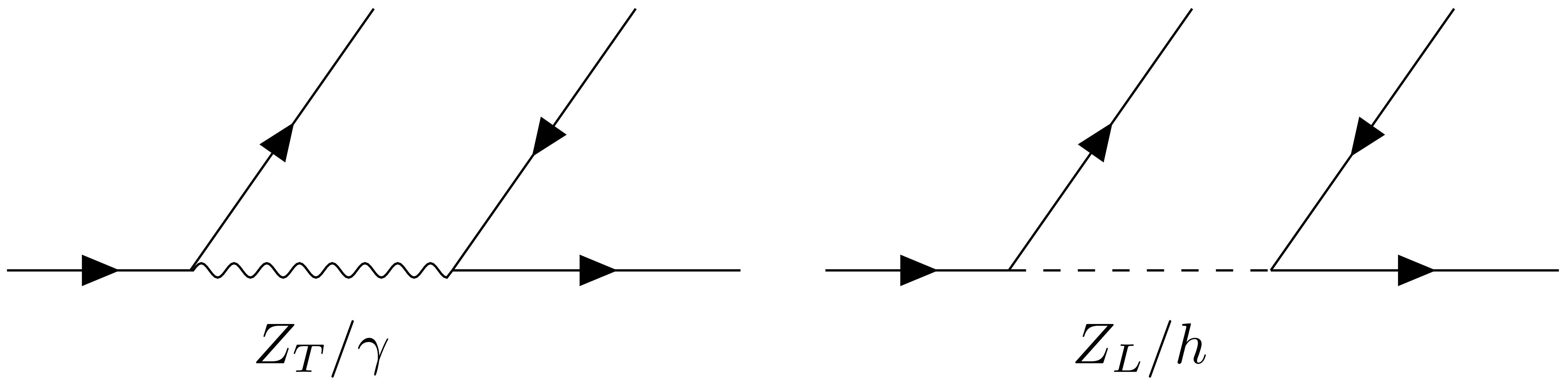


Models with quantum interference

- EW shower
  - Classical treatment

    Z. Nagy, E. Soper [0706.0017]

    J. Chen, T. Han, B. Tweedie [1611.00788]



- Simple toy model: N_f fermions charged under dark $U(1)$
  - $\mathscr{L}_{\rm dark} = \sum_i \bar{\chi}_i (i\partial\!\!\!/ - m_{\chi_i})\chi_i + \sum_{i,j} i g_{ij} \bar{\chi}_i A\!\!\!/' \chi_j - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} - \frac{1}{2} m_{A'}^2 A'_\mu A'^\mu$

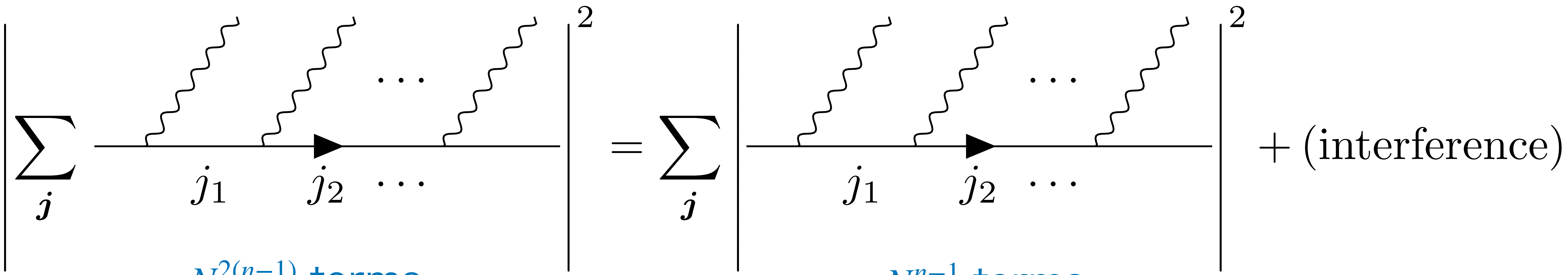
- Classical parton shower simulation can not take into account quantum interference effects in these models, but they can be phenomenologically important

Analytic treatment of interference effects

- A simple toy model with N_f flavors of fermions

$$\mathcal{L}_{\text{int}} = i\bar{\chi} G A' \chi \quad \text{with} \quad \chi = \begin{pmatrix} \chi_1 \\ \vdots \\ \chi_{N_f} \end{pmatrix} \; ; \; G = \begin{pmatrix} g & \cdots & g \\ \vdots & & \vdots \\ g & \cdots & g \end{pmatrix}$$

- For n gauge boson processes

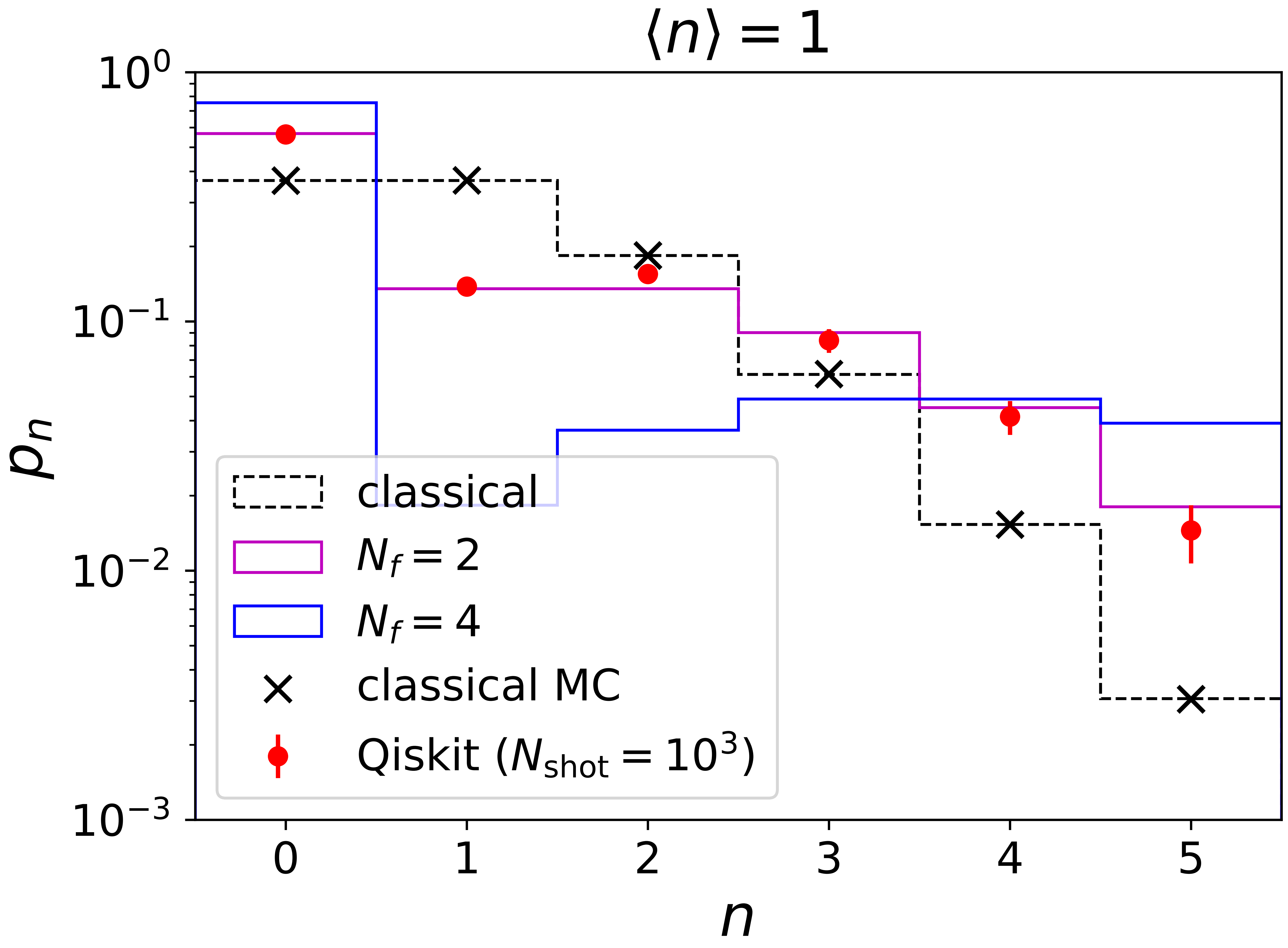


$N_f^{2(n-1)}$ terms

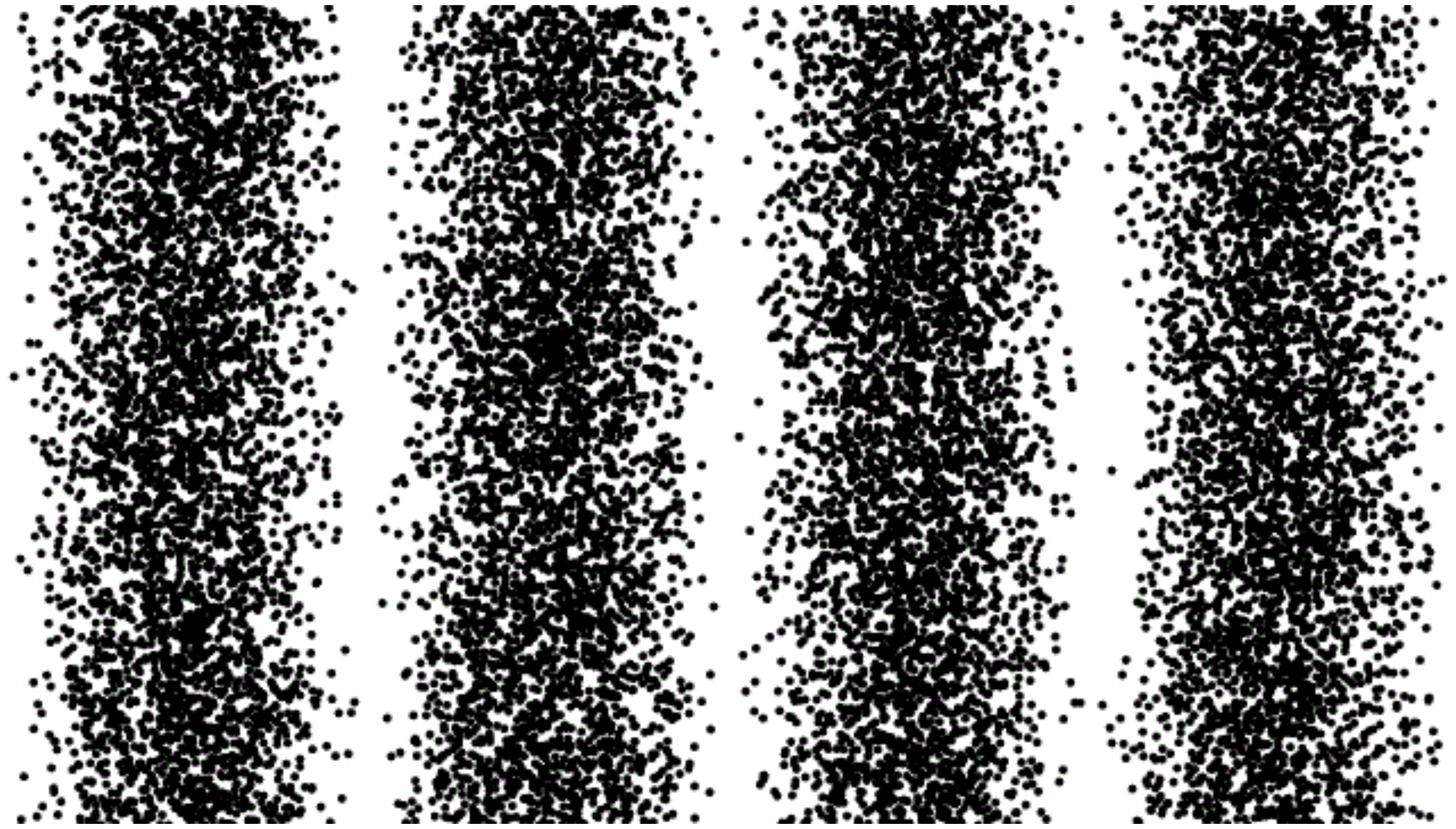
N_f^{n-1} terms

- Simple rescaling $\frac{p_n}{p_{n-1}} \to N_f \frac{p_n}{p_{n-1}}$ allows us to include the interference effects

Distribution of the number of emissions

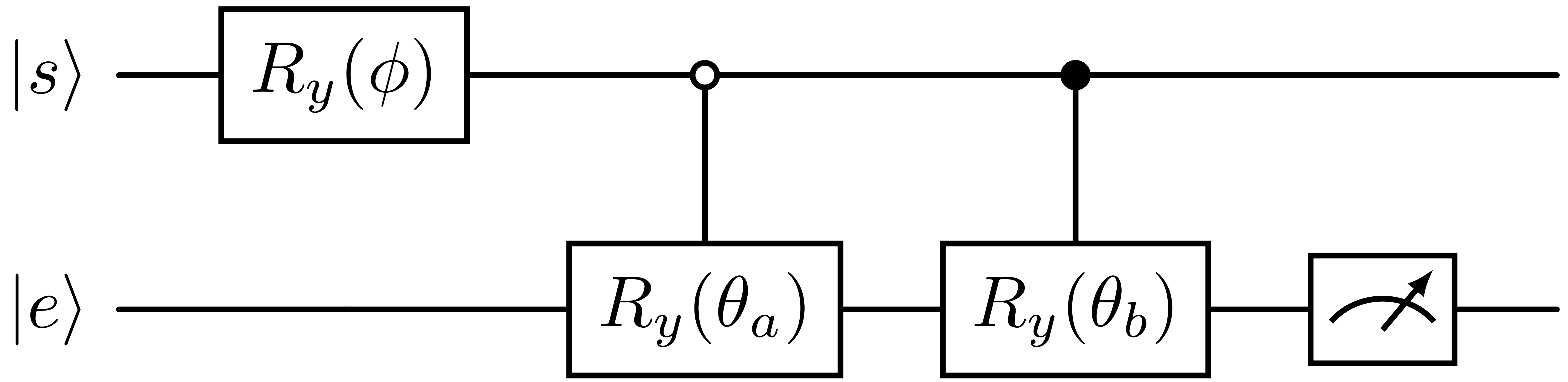


From classical to quantum simulation

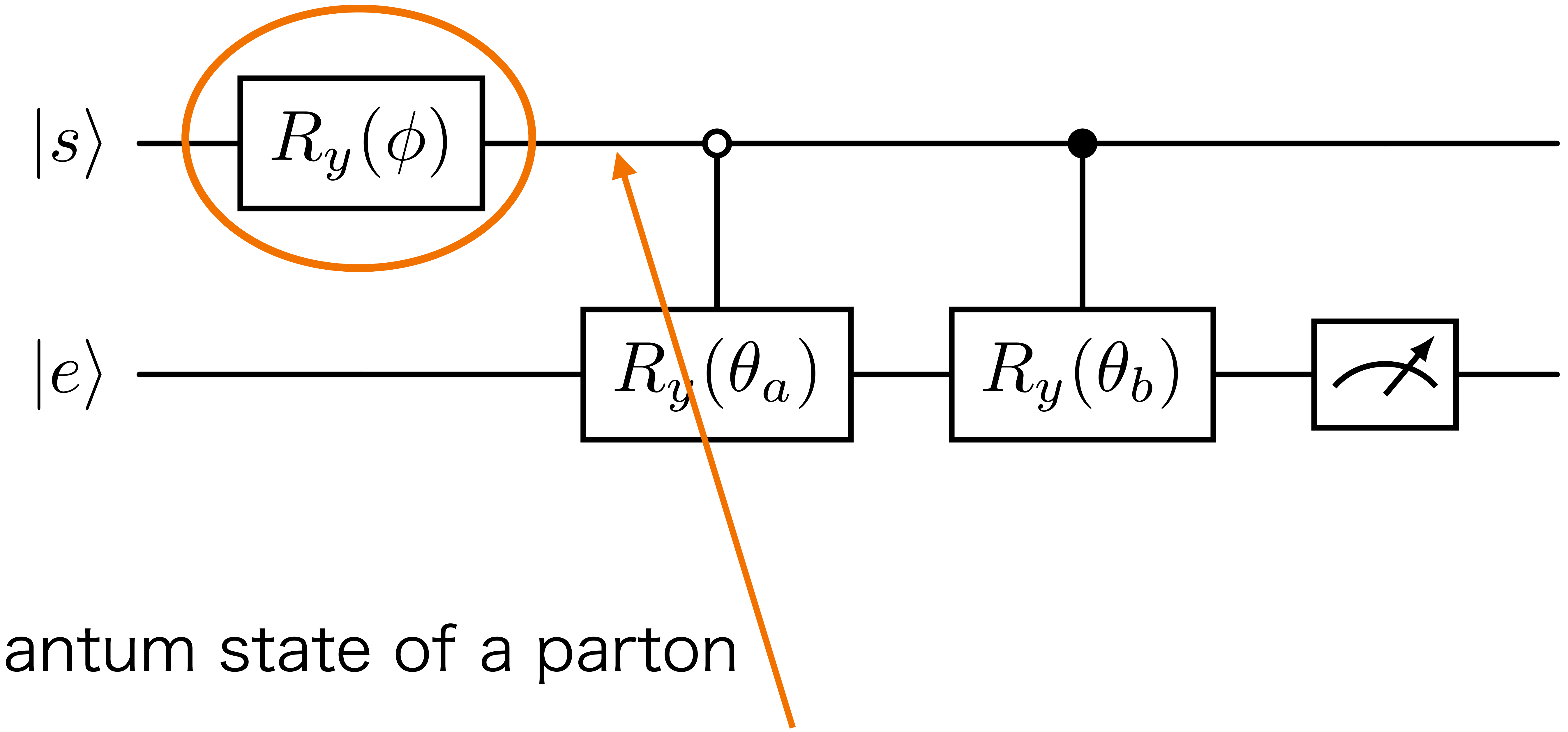


- The interference effect is a fundamental feature of the quantum mechanics
- Can we naturally include this effect in the numerical simulation by using superposition states in the quantum simulation?

Simplest two-flavor example



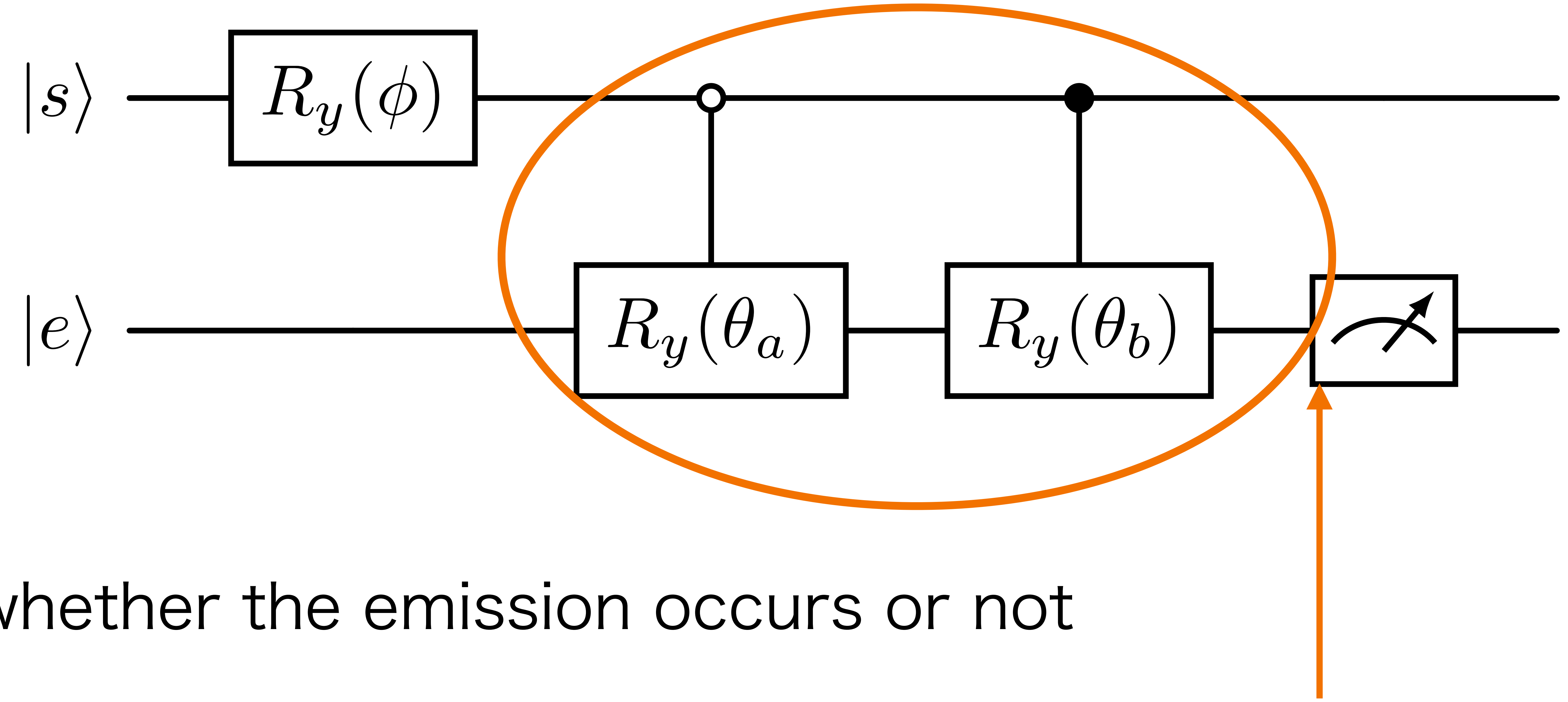
Simplest two-flavor example



- $|s\rangle$ stores a quantum state of a parton

$$|s\rangle = \begin{pmatrix} \cos\frac{\phi}{2} & -\sin\frac{\phi}{2} \\ \sin\frac{\phi}{2} & \cos\frac{\phi}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \cos\frac{\phi}{2}|a\rangle + \sin\frac{\phi}{2}|b\rangle$$

Simplest two-flavor example

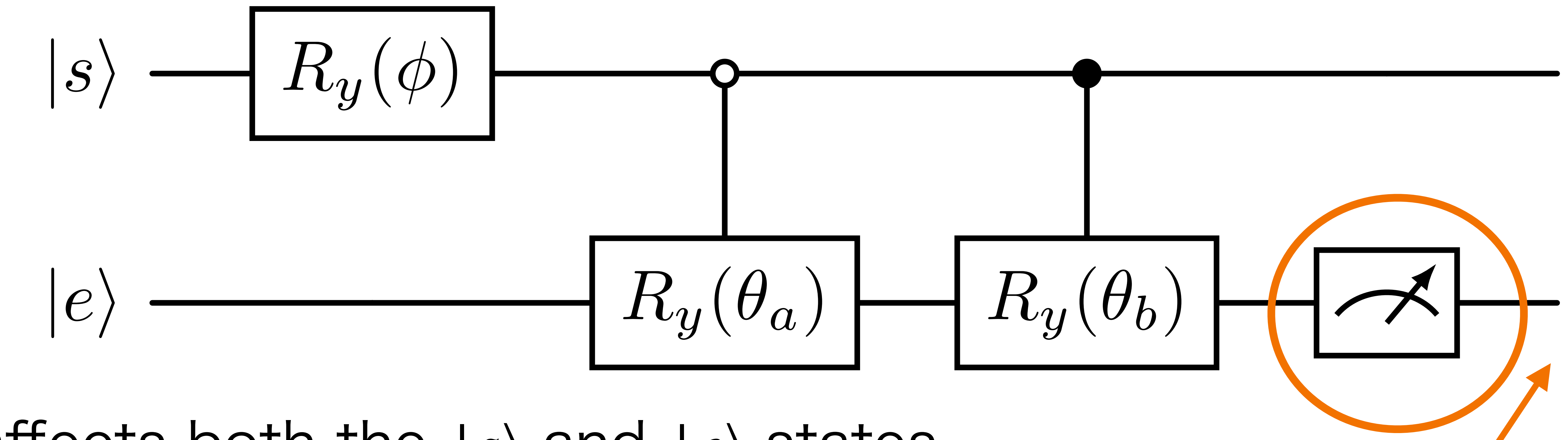


- $|e\rangle$ preserves whether the emission occurs or not

$$|\psi\rangle \equiv |s\rangle|e\rangle = \cos\frac{\phi}{2}|a\rangle\left(\cos\frac{\theta_a}{2}|0_e\rangle + \sin\frac{\theta_a}{2}|1_e\rangle\right) + \sin\frac{\phi}{2}|b\rangle\left(\cos\frac{\theta_b}{2}|0_e\rangle + \sin\frac{\theta_b}{2}|1_e\rangle\right)$$

- Emission probability from $|q\rangle$ $(q = a, b)$ - $p_q = \sin^2\frac{\theta_q}{2}$

Simplest two-flavor example



- Measurement affects both the $|s\rangle$ and $|e\rangle$ states

$$|\psi\rangle \equiv |s\rangle|e\rangle = \cos\frac{\phi}{2}|a\rangle\left(\cos\frac{\theta_a}{2}|0_e\rangle + \sin\frac{\theta_a}{2}|1_e\rangle\right) + \sin\frac{\phi}{2}|b\rangle\left(\cos\frac{\theta_b}{2}|0_e\rangle + \sin\frac{\theta_b}{2}|1_e\rangle\right)$$

$$\Rightarrow |\psi\rangle \propto \left(\cos\frac{\phi}{2}\cos\frac{\theta_a}{2}|a\rangle + \sin\frac{\phi}{2}\cos\frac{\theta_b}{2}|b\rangle\right)|0_e\rangle \qquad (e=0)$$

$$\Rightarrow |\psi\rangle \propto \left(\cos\frac{\phi}{2}\sin\frac{\theta_a}{2}|a\rangle + \sin\frac{\phi}{2}\sin\frac{\theta_b}{2}|b\rangle\right)|1_e\rangle \qquad (e=1)$$

Towards sampling: veto method

- We judge if emission occurs in $t_j < t < t_j + \Delta t$ and sample x according to

$$\Delta \mathscr{P} \simeq R(t_j)\,\Delta t \quad \text{and} \quad R(t_j) \simeq \frac{1}{t_j}\int_{x_{\min}(t_j)}^{x_{\max}(t_j)} dx\, \frac{\alpha(t_j, x)}{2\pi} P(x)$$

- The veto method for sampling based on a complicated distribution $f(x)$

1) Prepare over-estimated quantities

$$f^{\text{over}}(x) \geq f(x) \quad \text{with} \quad \int_{x_{\min}^{\text{over}}}^{x_{\max}^{\text{over}}} dx\, f^{\text{over}}(x) = 1$$

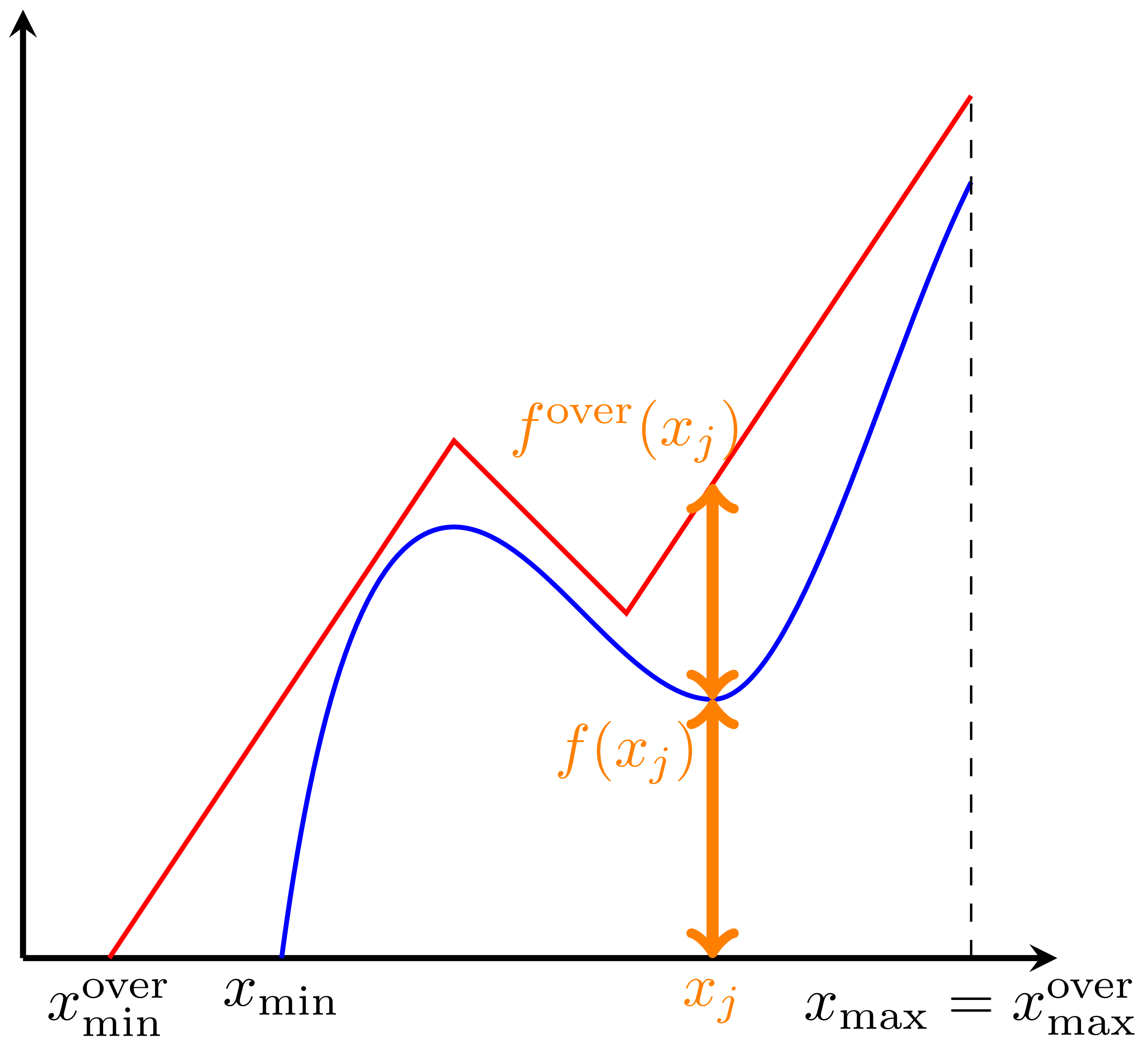
$$\left[x_{\min}^{\text{over}}, x_{\max}^{\text{over}}\right] \supseteq \left[x_{\min}, x_{\max}\right]$$

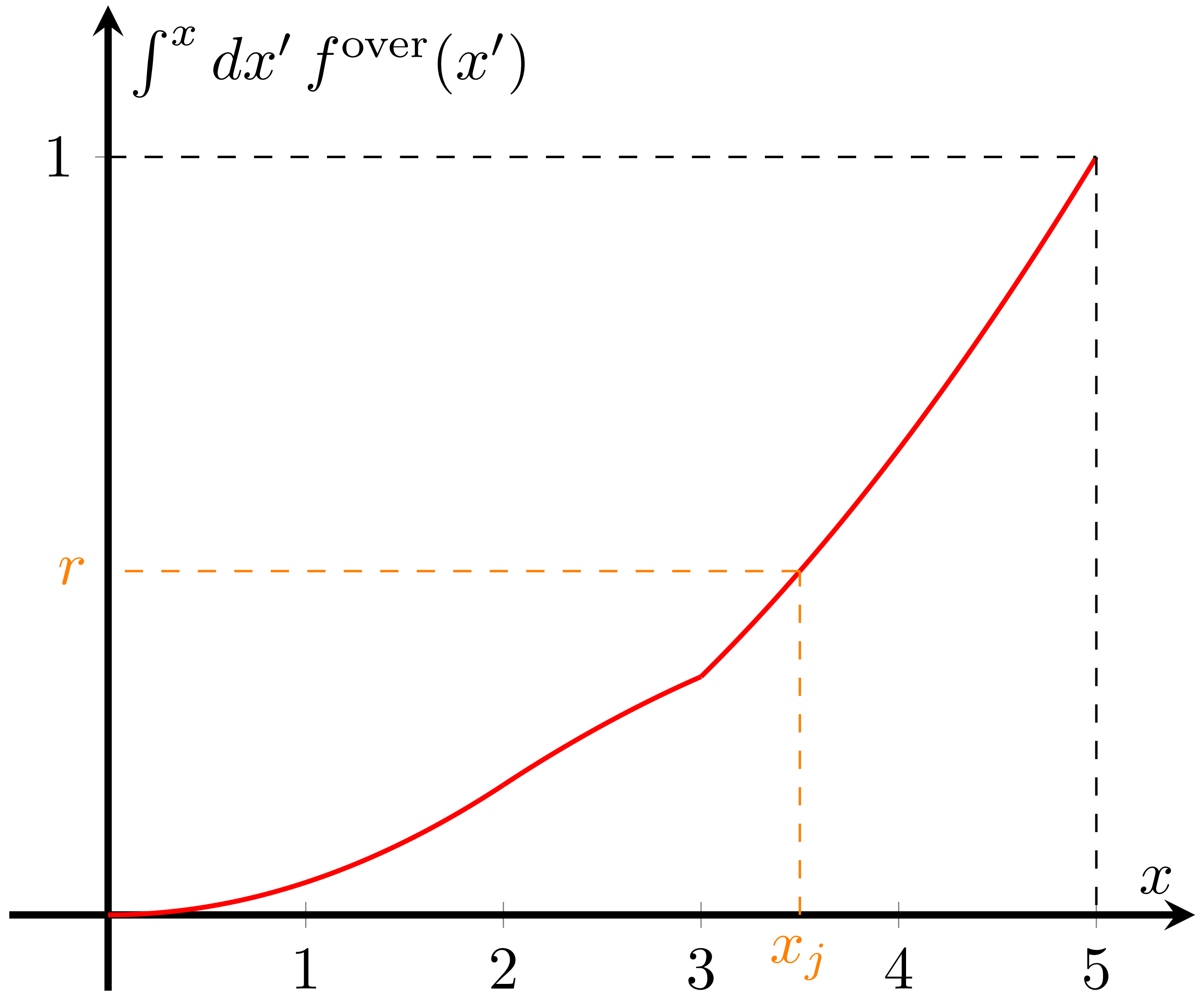
2) Sample x_j according to $f^{\text{over}}(x)$

Solve $\int_{x_{\min}^{\text{over}}}^{x_j} dx' f^{\text{over}}(x') = r \in [0,1)$

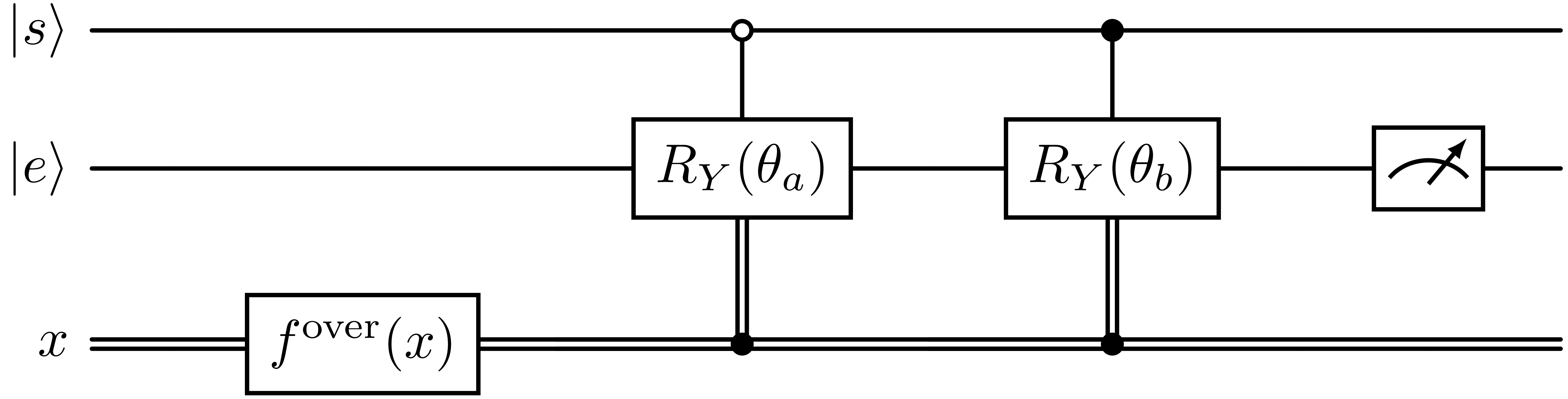
3) Veto (= conclude no emission) if

$x_j \notin [x_{\min}, x_{\max}]$ or $f(x_j)/f^{\text{over}}(x_j) < r' \in [0,1)$

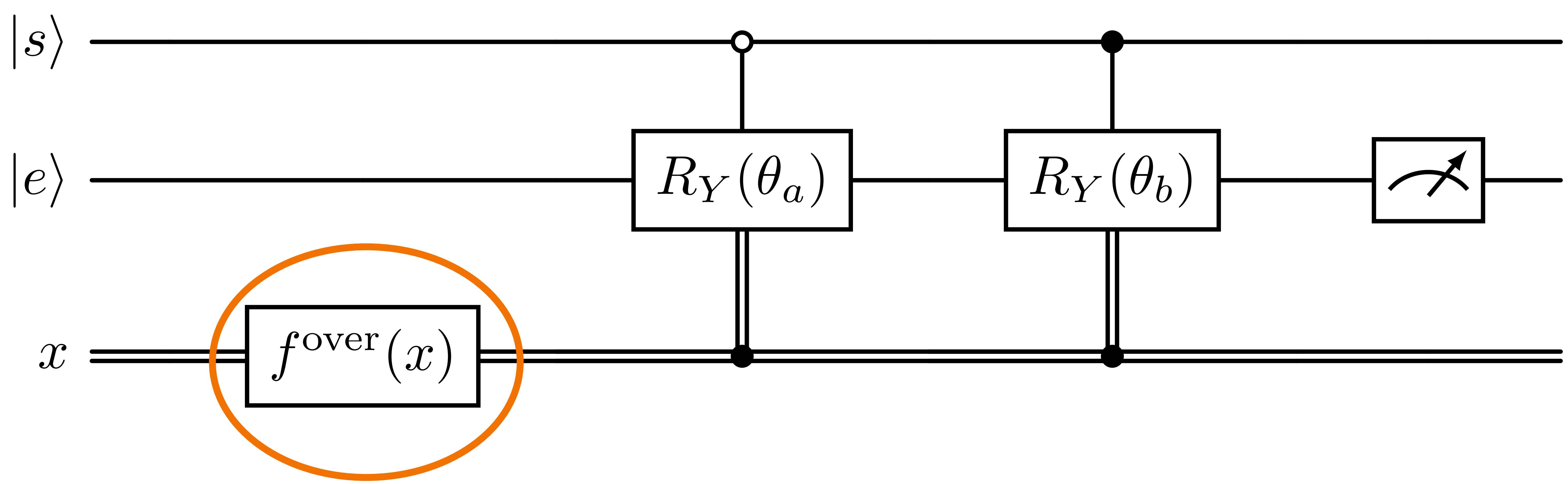




Two-flavor simulation with sampling

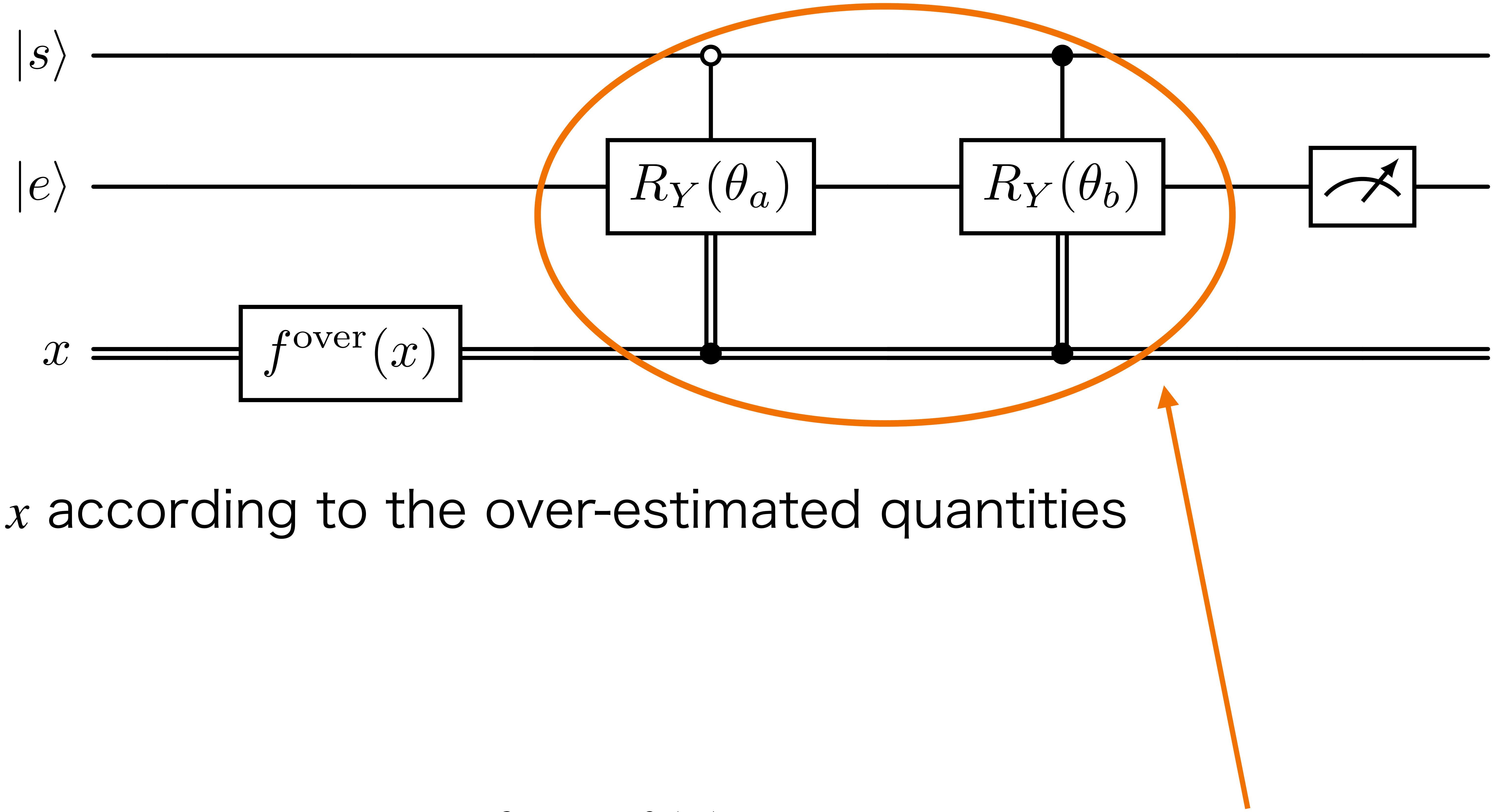


Two-flavor simulation with sampling



- Sampling of x according to the over-estimated quantities

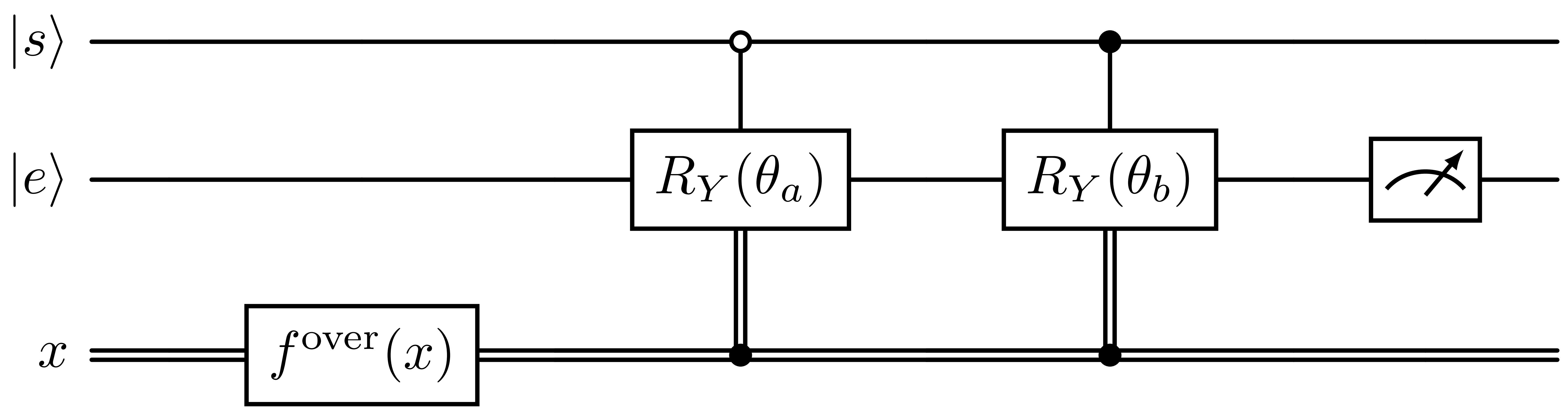
Two-flavor simulation with sampling



- Sampling of x according to the over-estimated quantities

- State-dependent veto with $\sin^2 \frac{\theta_q}{2} = \frac{f_q(x_j)}{f^{\text{over}}(x_j)}$ for $|s\rangle = |q\rangle \ \ (q = a, b)$

Two-flavor simulation with sampling

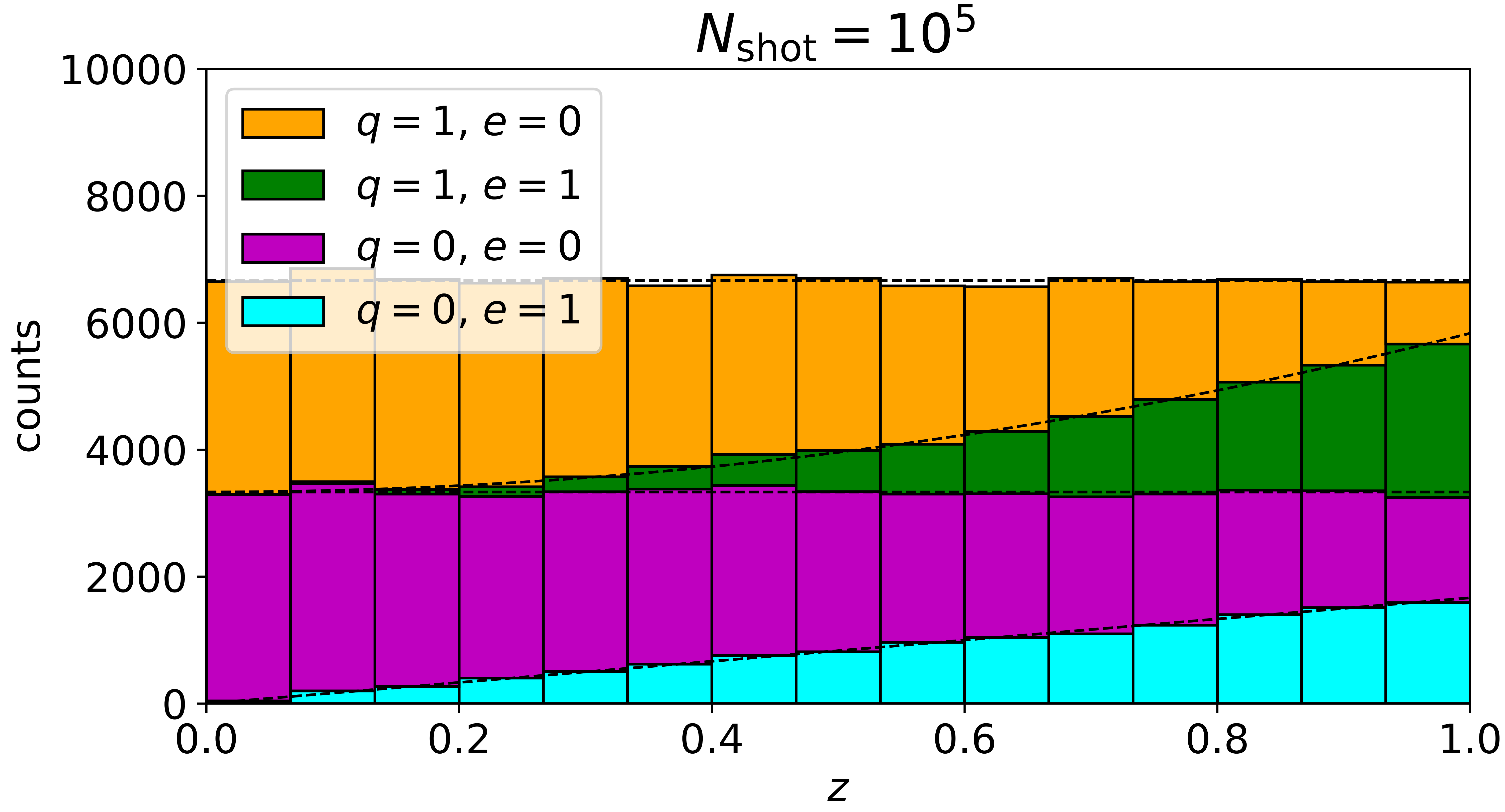


- Sampling of x according to the over-estimated quantities

Veto procedure allows to use state-independent $\mathbf{f^{over}(x)}$ for sampling as far as $\mathbf{f^{over}(x) \geq f_a(x), f_b(x)}$

- State-dependent veto with $\sin^2 \frac{\theta_q}{2} = \frac{f_q(x_j)}{f^{\rm over}(x_j)}$ for $|s\rangle = |q\rangle \;\; (q = a, b)$

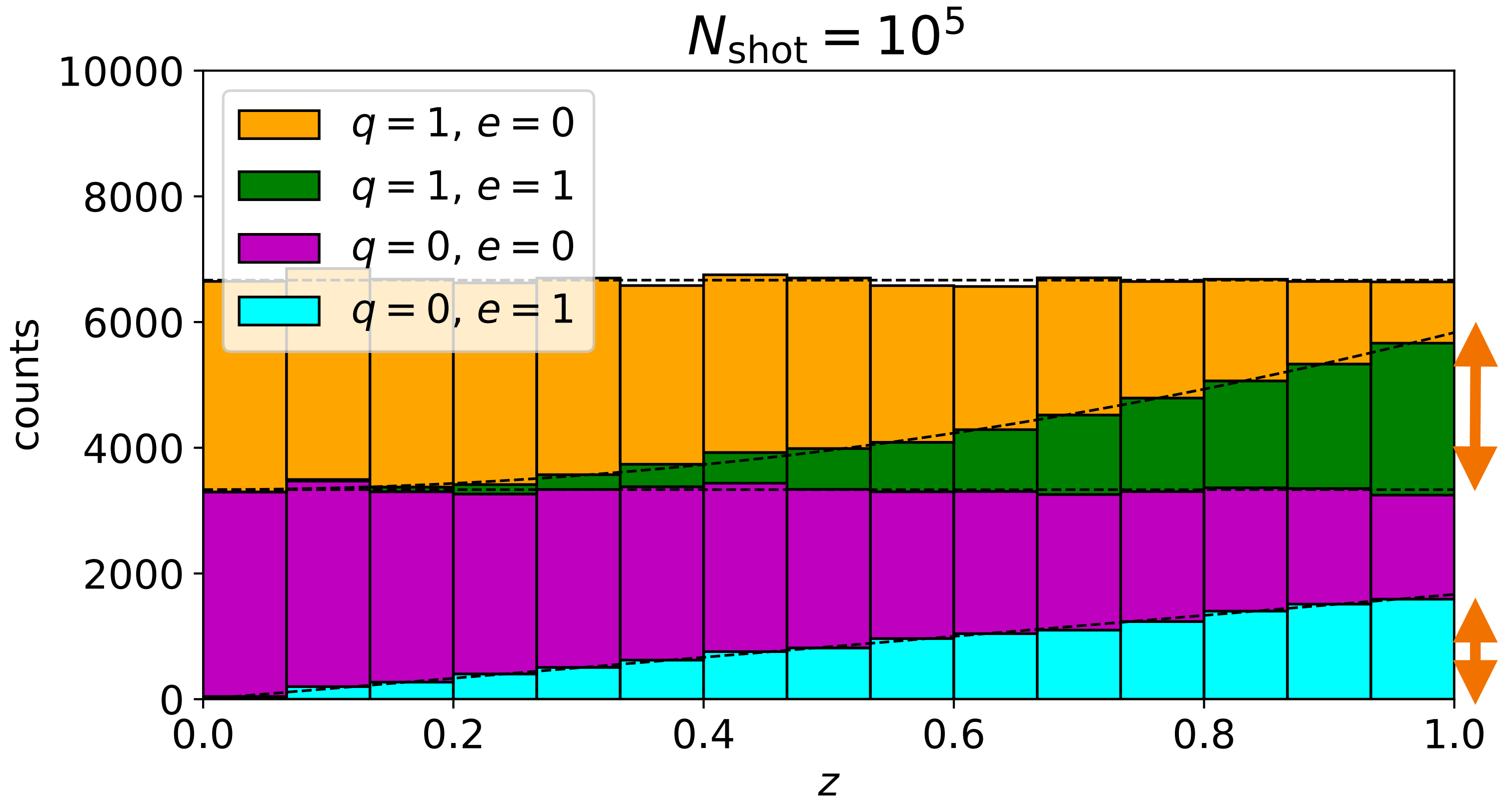
Numerical simulation by Qiskit

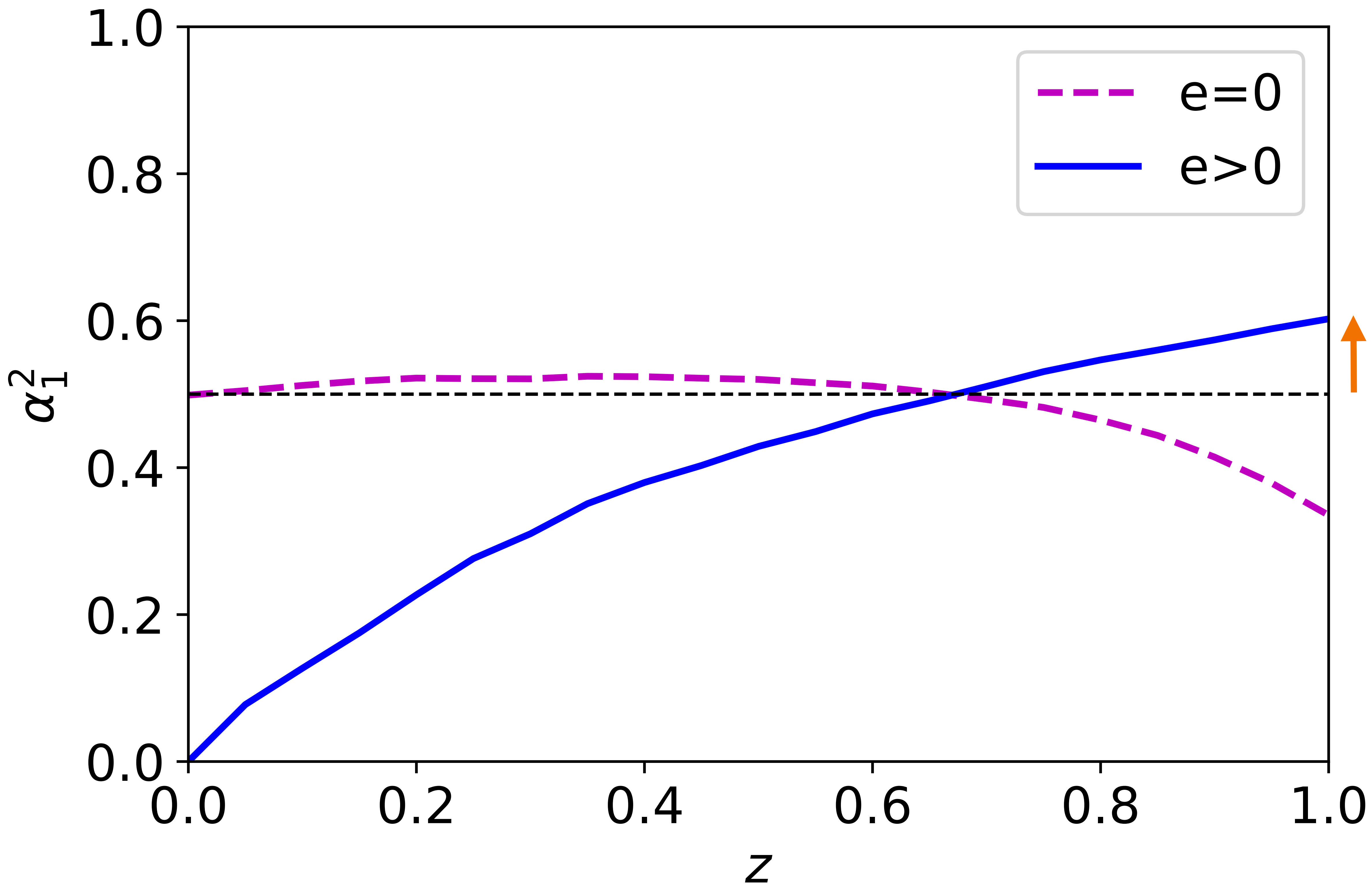


$$f_a(z) = \frac{1}{2}z \ , \ f_b(z) = \frac{3}{4}z^2 \text{ and } |s\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$$

Numerical simulation by Qiskit

- $|s\rangle = \alpha_0 |a\rangle + \alpha_1 |b\rangle$ after meas. of $|e\rangle$



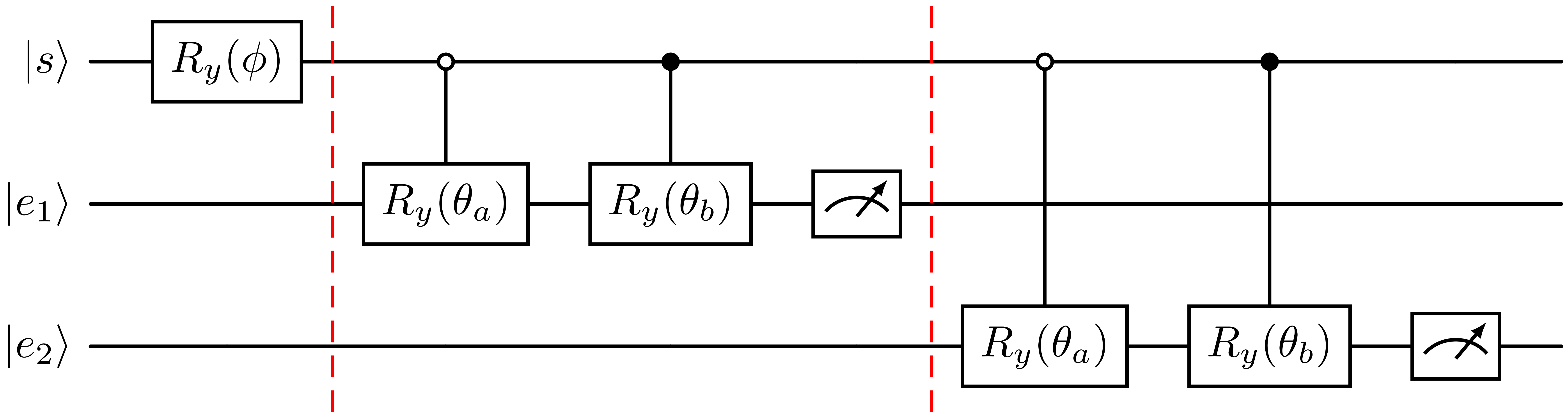


- Analytically / Numerically checked quantum state evolution is OK

  up to $O\left(\Delta\mathscr{P}_q^2\right)$ - Require $\Delta\mathscr{P}_a$, $\Delta\mathscr{P}_b \ll 1$

Quantum interference effect

- $(N = 2)$-step simulation starting from $|s\rangle = c_\phi |a\rangle + s_\phi |b\rangle$



- "Classical" anticipation

$$p^{(N=1)}_{e=1} = c_\phi^2 \Delta\mathscr{P}_a + s_\phi^2 \Delta\mathscr{P}_b$$

$$p^{(N=2)}_{e_1=e_2=1} = \left(p^{(N=1)}_{e=1}\right)^2$$

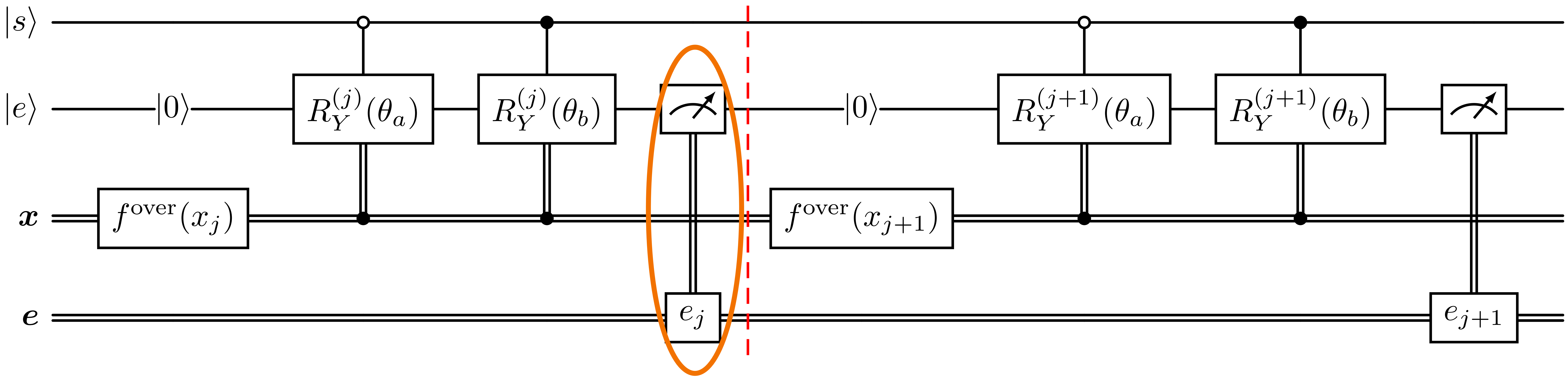
- Quantum result

$$p^{(N=1)}_{e=1} = c_\phi^2 \Delta\mathscr{P}_a + s_\phi^2 \Delta\mathscr{P}_b$$

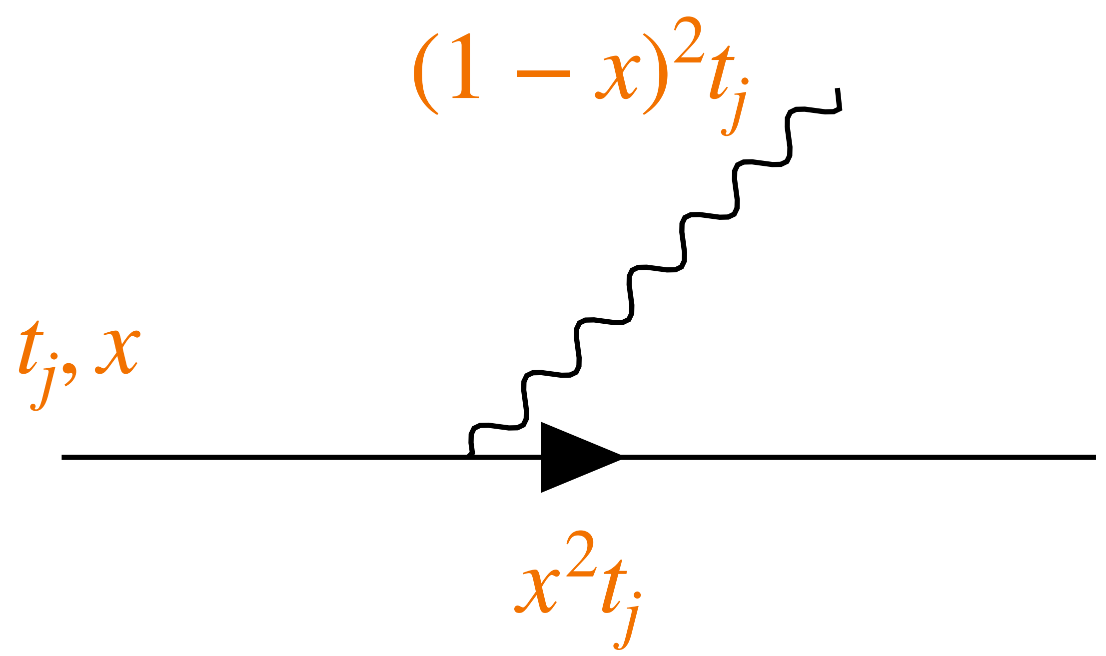
$$p^{(N=2)}_{e_1=e_2=1} = c_\phi^2 \Delta\mathscr{P}_a^2 + s_\phi^2 \Delta\mathscr{P}_b^2 \neq \left(p^{(N=1)}_{e=1}\right)^2$$

Multi-step simulation with kinematics

- Discretize $t \in [\mu_{\rm IR}^2, E_0^2]$ to N-steps as $t_0 = E_0^2 > t_1 > t_2 > \cdots > t_N = \mu_{\rm IR}^2$



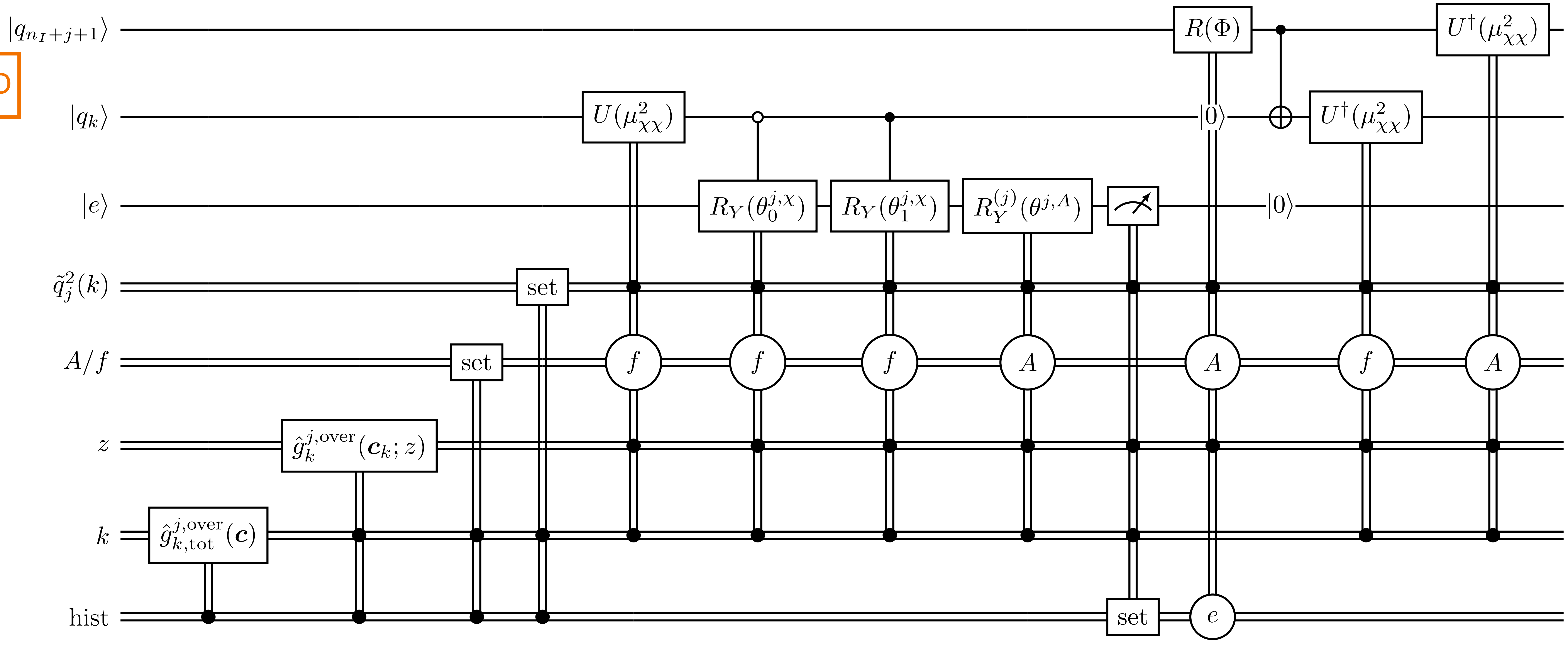
- Need mid-circuit measurement to track the preceding dynamics with full kinematics



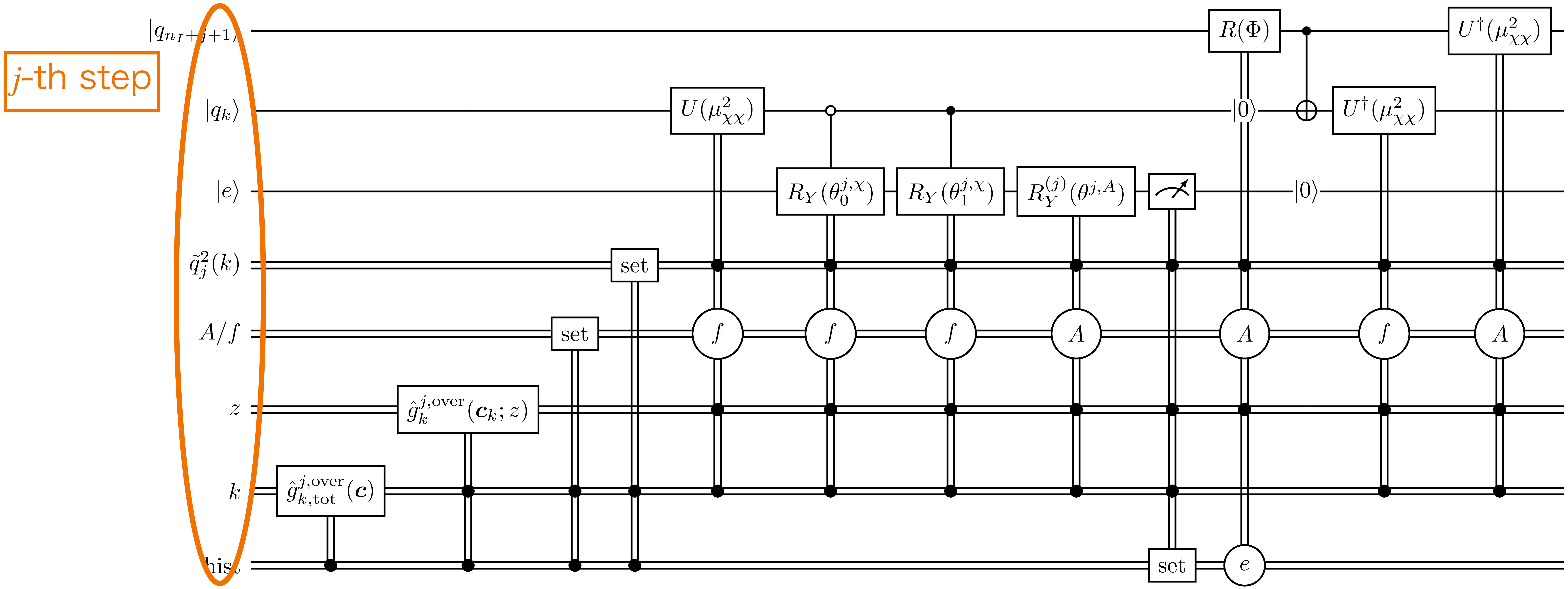
1) Add new parton
2) Vituality jump

Quantum Veto Parton Shower

j-th step

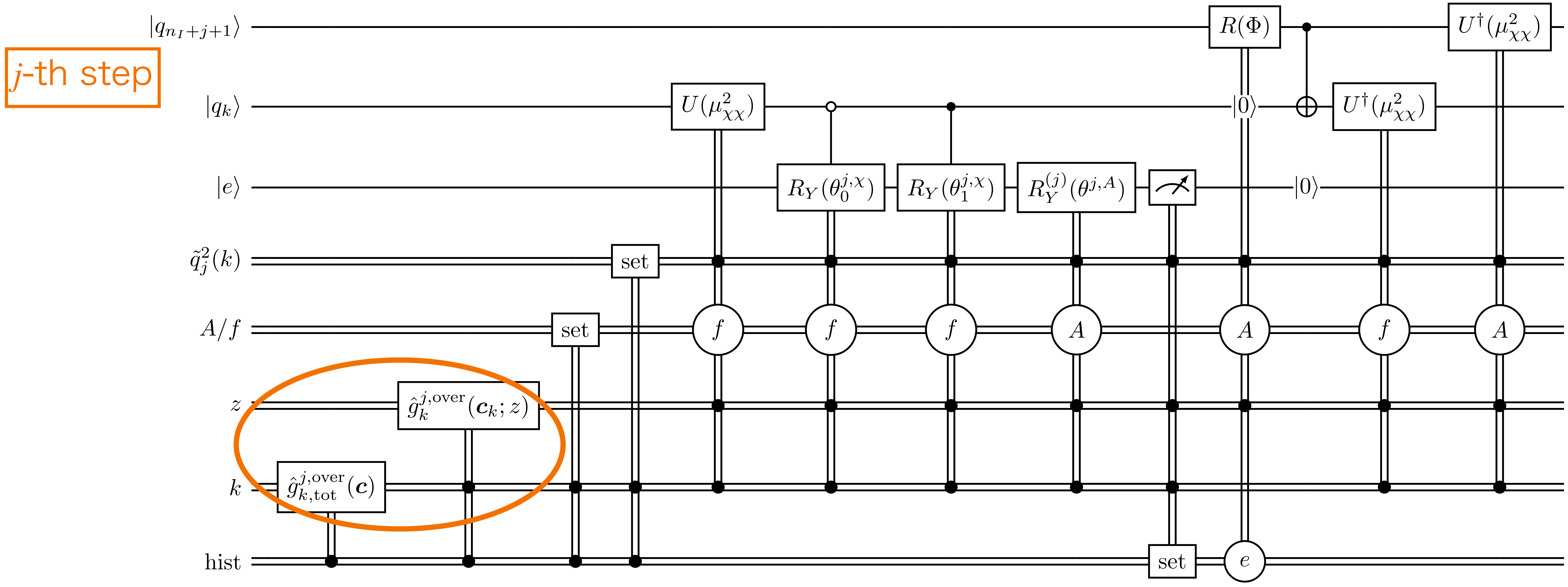


Quantum Veto Parton Shower



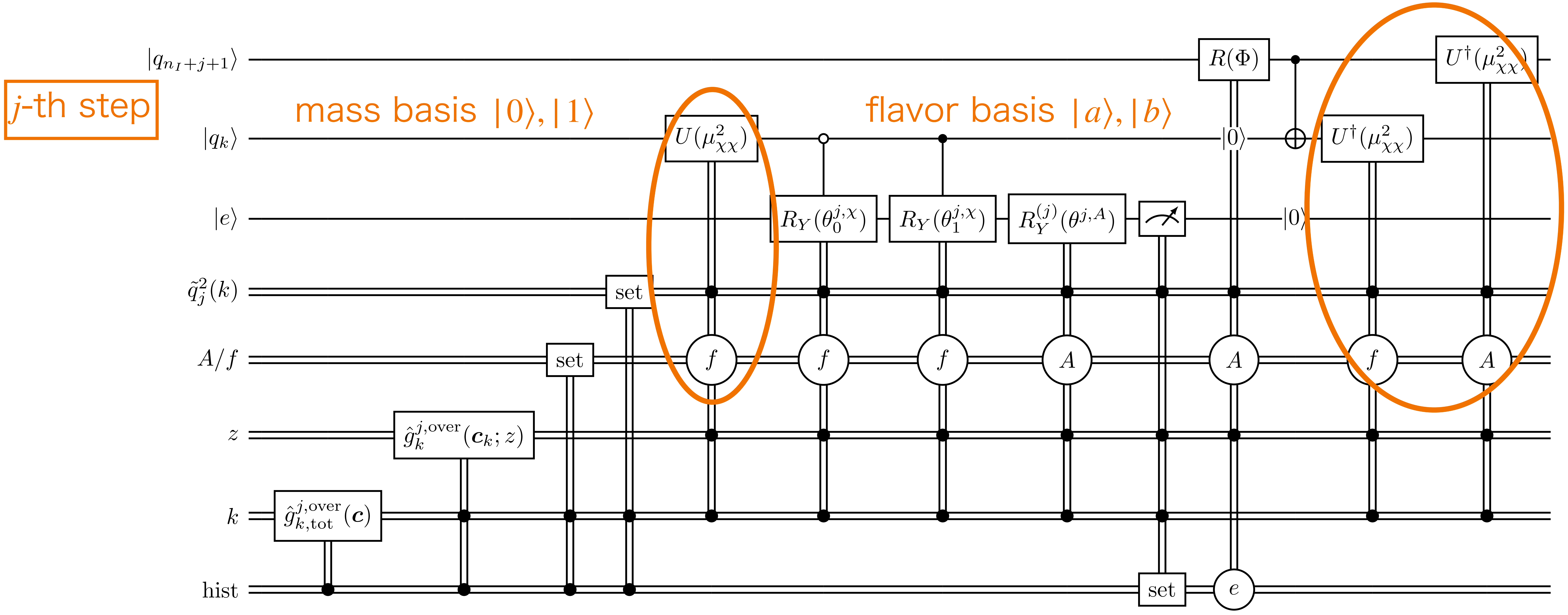
- Particle register $|q_k\rangle$ for each parton k stores fermion flavors in $\lceil \log_2 N_f \rceil$ qubits
- Virtuality of each patron $\tilde{q}_j^2(k)$, whether it is a fermion / gauge boson, stored in classical bits
- Emission history is also stored in classical bits

Quantum Veto Parton Shower



- Sampling of k (a candidate parton that undergoes emission) and z (energy fraction)
  - Sampling of k can be done classically again thanks to the over-estimated quantities
  - Candidate splitting topology and kinematics is fixed

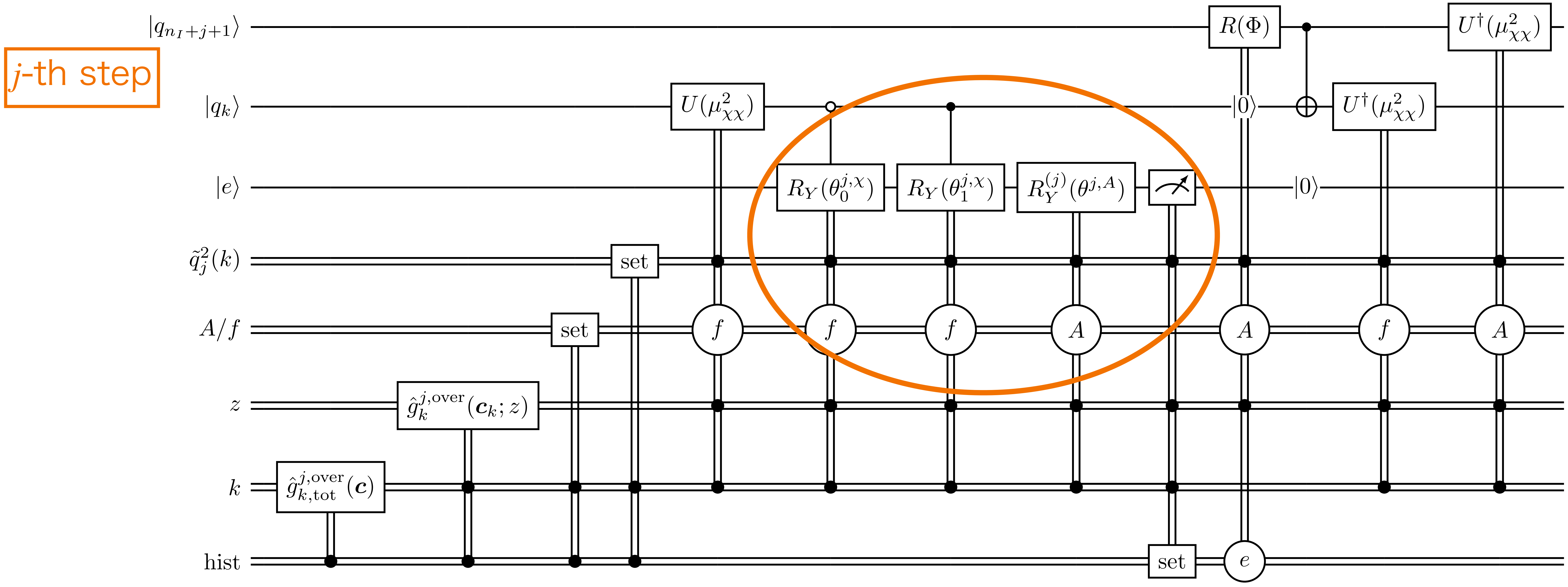
Quantum Veto Parton Shower



- Basis rotation of fermion (if necessary)
  - Due to the RGE flow, the rotation angle is scale/kinematics-dependent
  - Suitable choice of the RG scale is process dependent

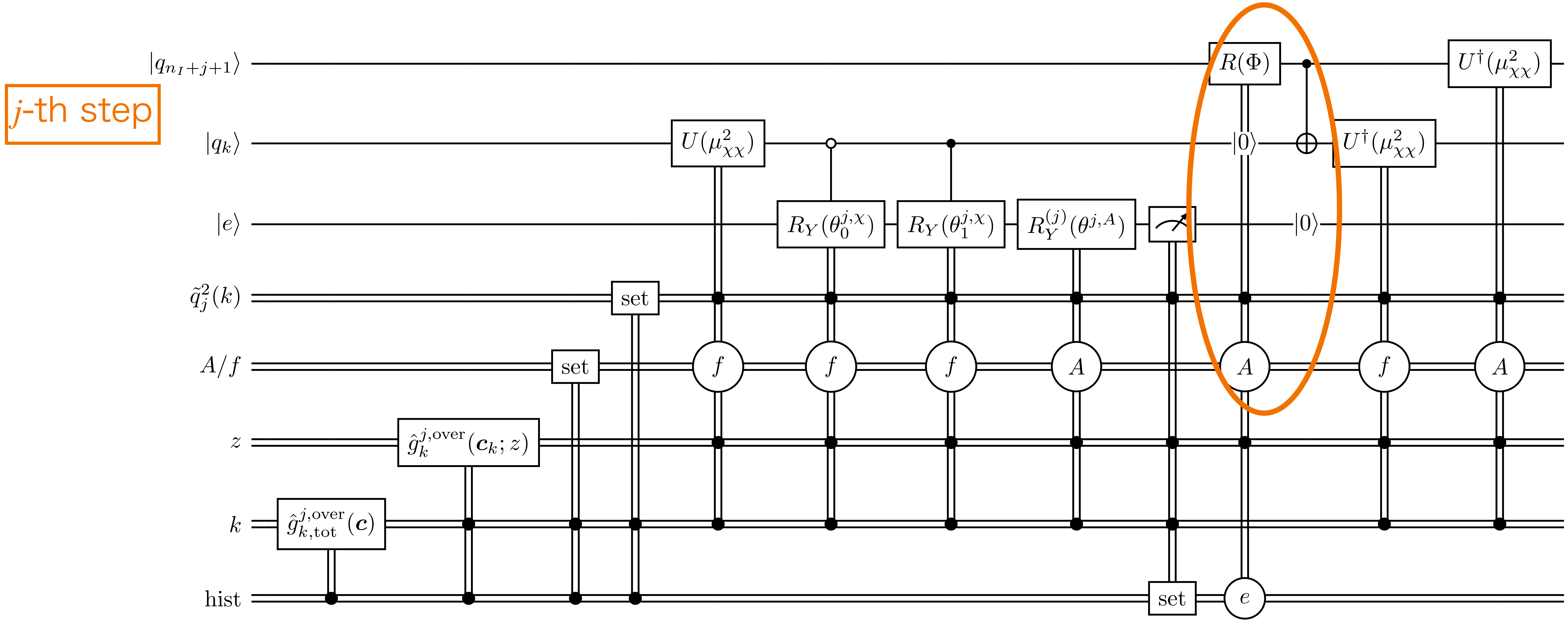
Herwig++ Physics and Manual [0803.0883]

Quantum Veto Parton Shower



- Veto and determine whether the emission occurs through the mid-circuit measurement

Quantum Veto Parton Shower



- If emission occurs, state update is necessary
  - k = fermion, add a new gauge boson
  - k = gauge boson, generate an **entangled** state $|q_k\rangle|q_{\mathrm{new}}\rangle = \dfrac{1}{\alpha_a^2 + \alpha_b^2}\left(\alpha_a|a\rangle|a\rangle + \alpha_b|b\rangle|b\rangle\right)$

Numerical results of QVPS

- Summary of the setup

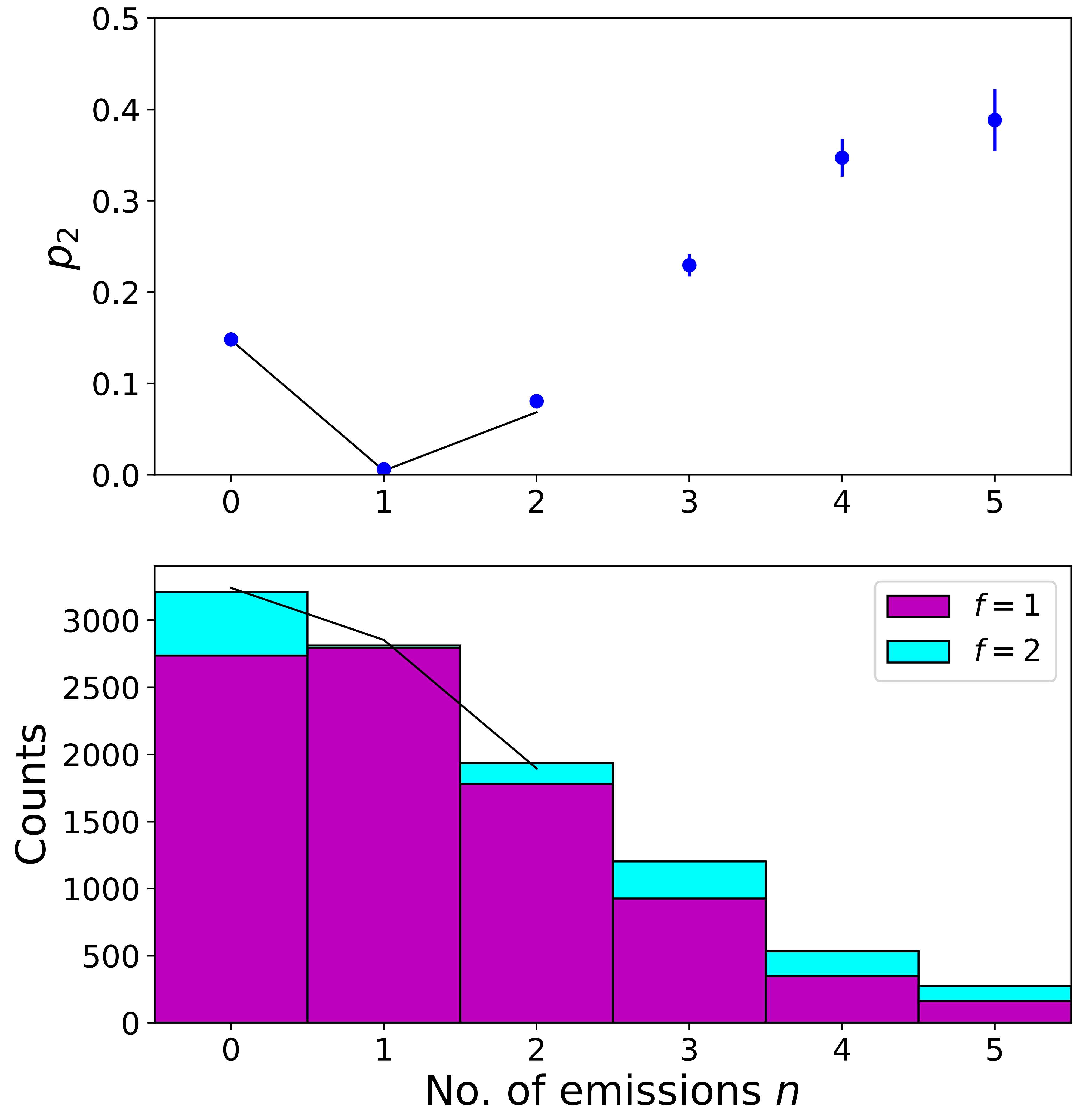
$$|s\rangle = |1\rangle \equiv \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$$

$$\alpha_f(\mu) = \frac{\alpha_f^0}{\beta_0 \ln(\mu^2/\Lambda_D^2)} \quad (\alpha_0^a = 0.5,\ \alpha_0^b = 2)$$

$$\beta_0 = \frac{33 - 2N_f}{4\pi} \quad (N_f = 2)$$

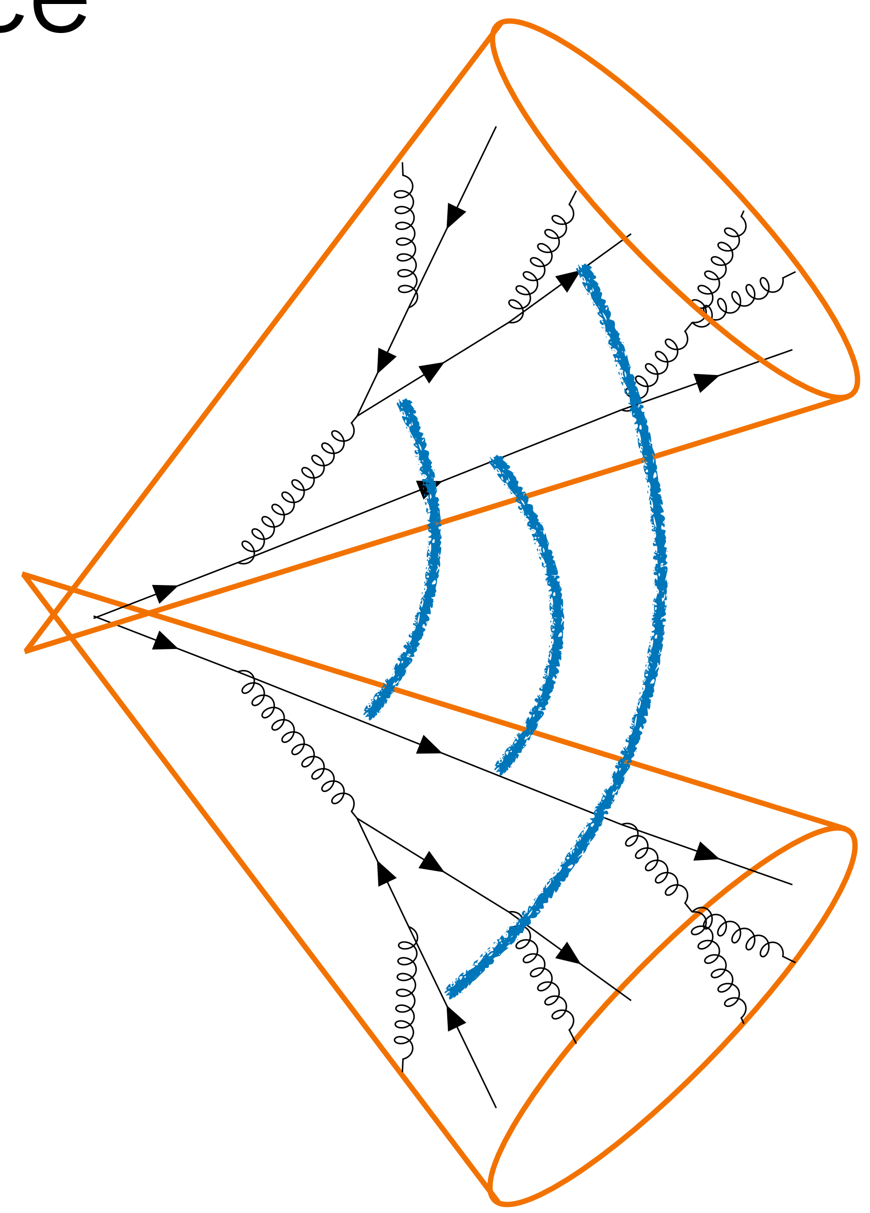
- Some analytic result

$$p_2(n) = \frac{\left(\sqrt{N_a(n)} - \sqrt{N_b(n)}\right)^2}{2\left(N_a(n) + N_b(n)\right)}$$



Future directions

- Construct more efficient algorithms
  - $\Delta\mathscr{P}_a$, $\Delta\mathscr{P}_b \ll 1$ enforces fine mesh of t
  - Directly sampling t with veto
  - Gate cost $O(N) \to O(\langle n \rangle)$ Work in progress
- Treatment of soft interference
  - Global entanglement
  - Quantum treatment of history
- Next-to-leading logarithms
- Noise mitigation

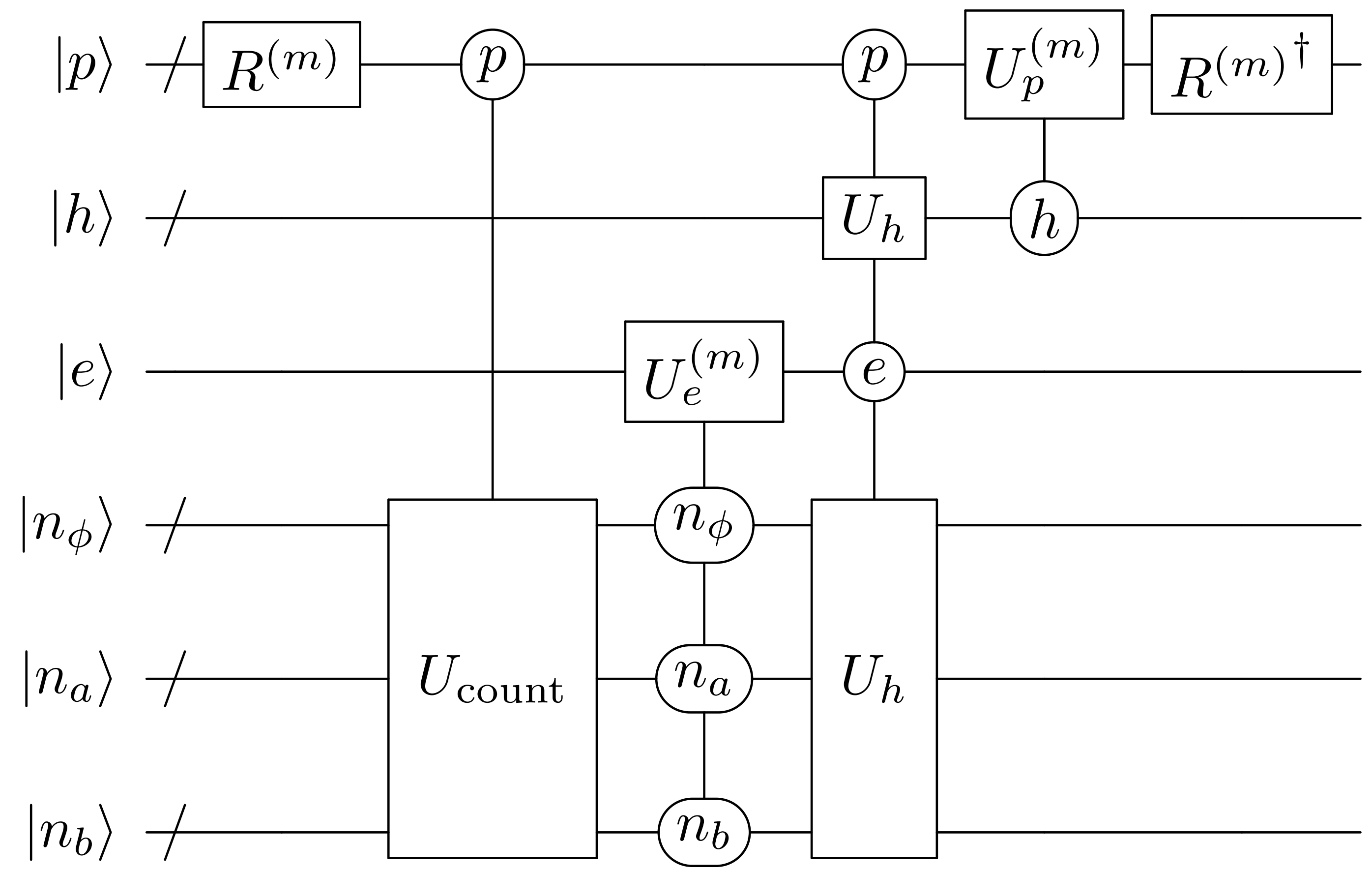


- Quantum resources required

Register	Purpose	2 flavors
$\vert q\rangle$	Particle state	$N + n_I$
$\vert e\rangle$	Did emission happen?	1

Element	Purpose	Gate Cost
$U(\mu^2)$	Flavor rotation	$2N$
$RY(\theta)$	Emission	$4N$
$R(\Phi)$	Particle update	$2N$

- Emission history in a qubit register



Bauer, et al. [1904.03196]

Conclusion

Problem

A non-trivial "flavor" structure could induce quantum interference effects in the multi-emission processes, which cannot be tracked by the classical parton shower algorithm

What we did

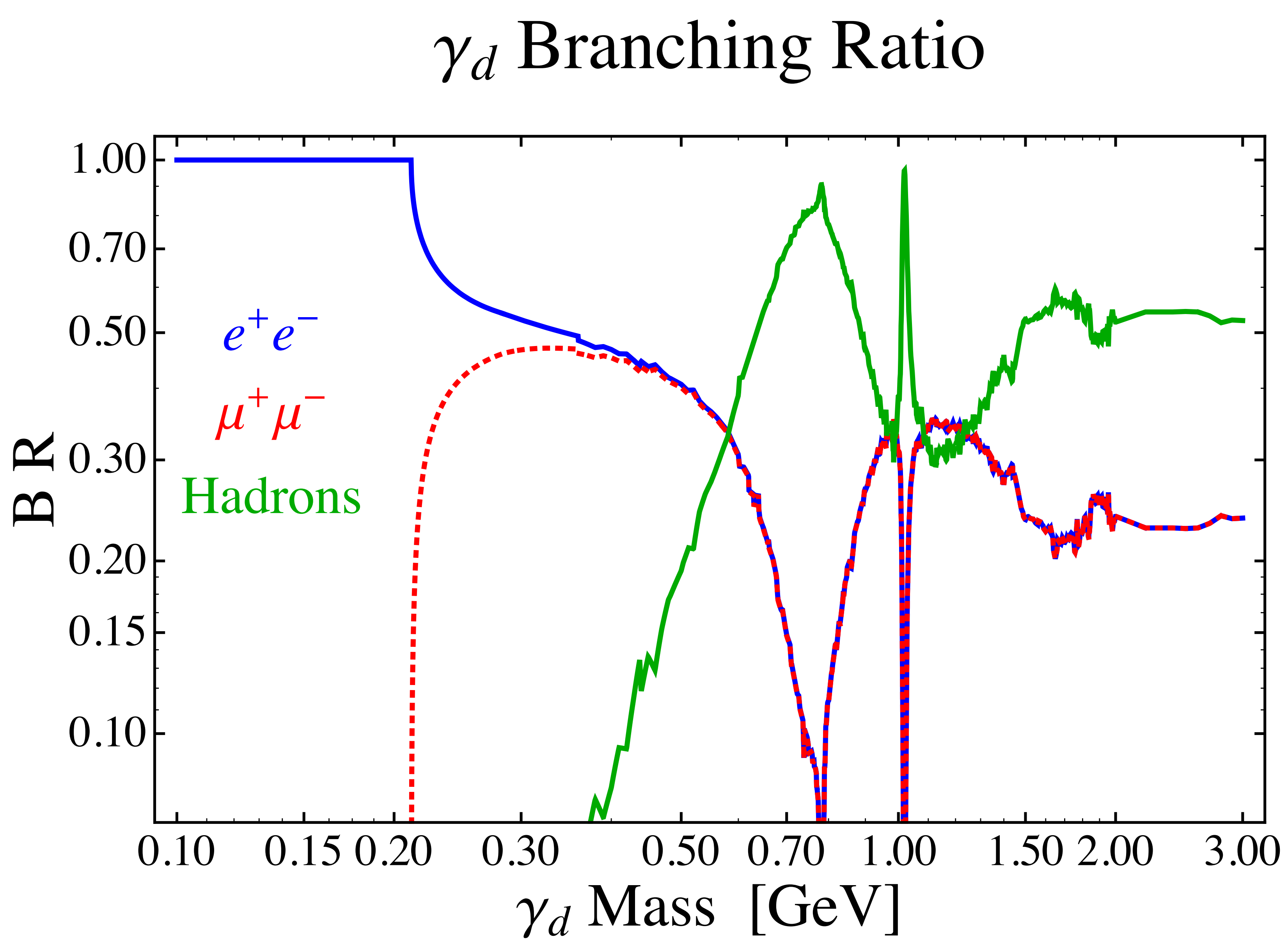
1. Constructed a quantum algorithm to simulate multi-emission processes, taking into account quantum interference and kinematical effects
2. Demonstrated the phenomenological implications based on a toy model

- Possible future directions include the optimization of the quantum circuit, inclusion of soft interference and next-leading order effects, noise mitigation, and more.

- Thank you for your interest!

Phenomenology example: lepton jets

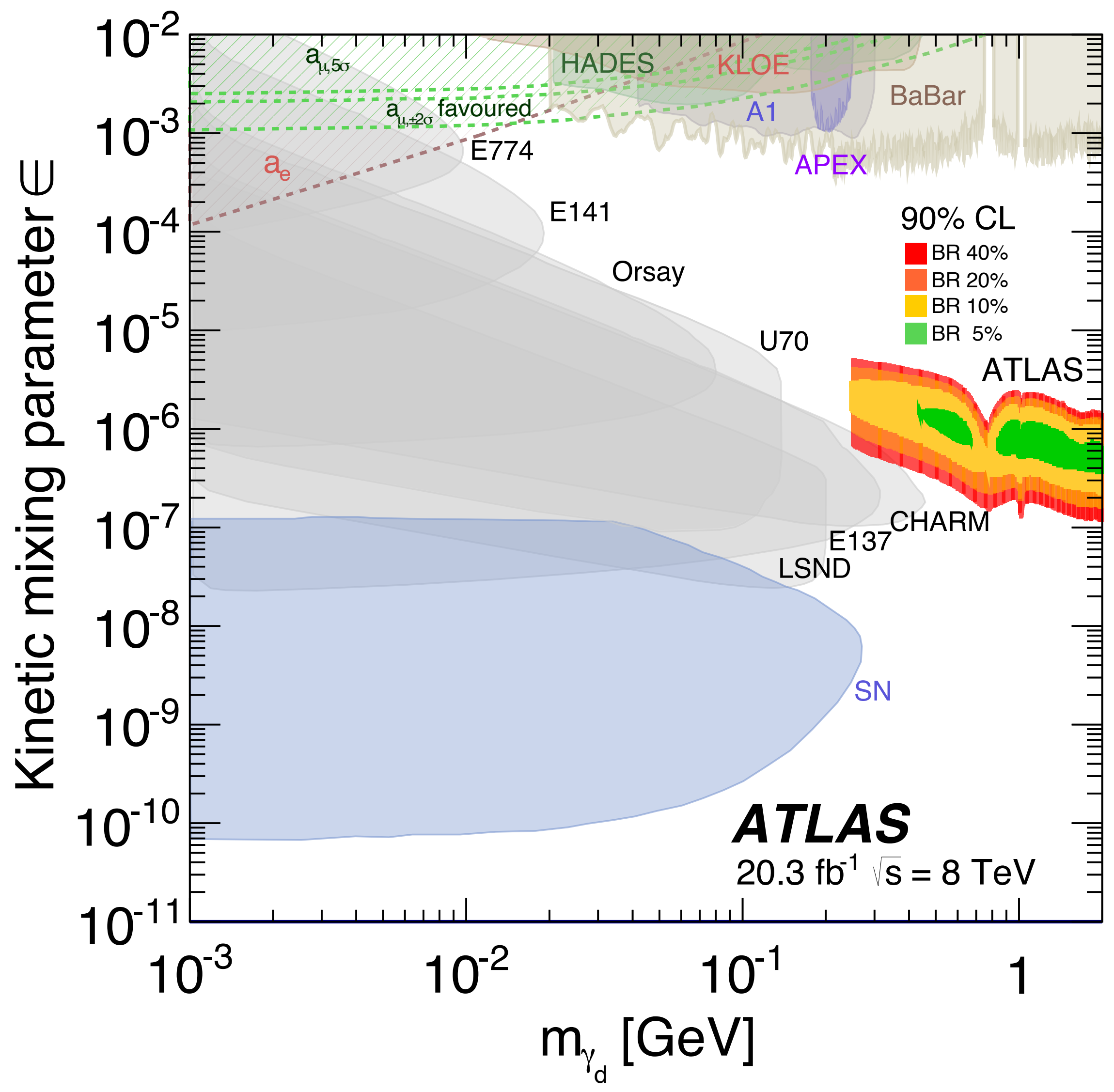
- Observe A' decay products from $pp \to \bar{\chi}\chi + nA'$
  - A' decay through kinetic mixing



A. Falkowski+ [1002.2952]

- "Lepton jets" for $m_{A'} \lesssim$ GeV

C. Cheung+ '09, P. Meade+ '09, A. Falkowsk+ '10



ATLAS [1212.5409]
ATLAS [1409.0746]

- Cuts on lepton multiplicity eg) $\geq$ 4 muons

- Interference effect on number distribution of emissions matters