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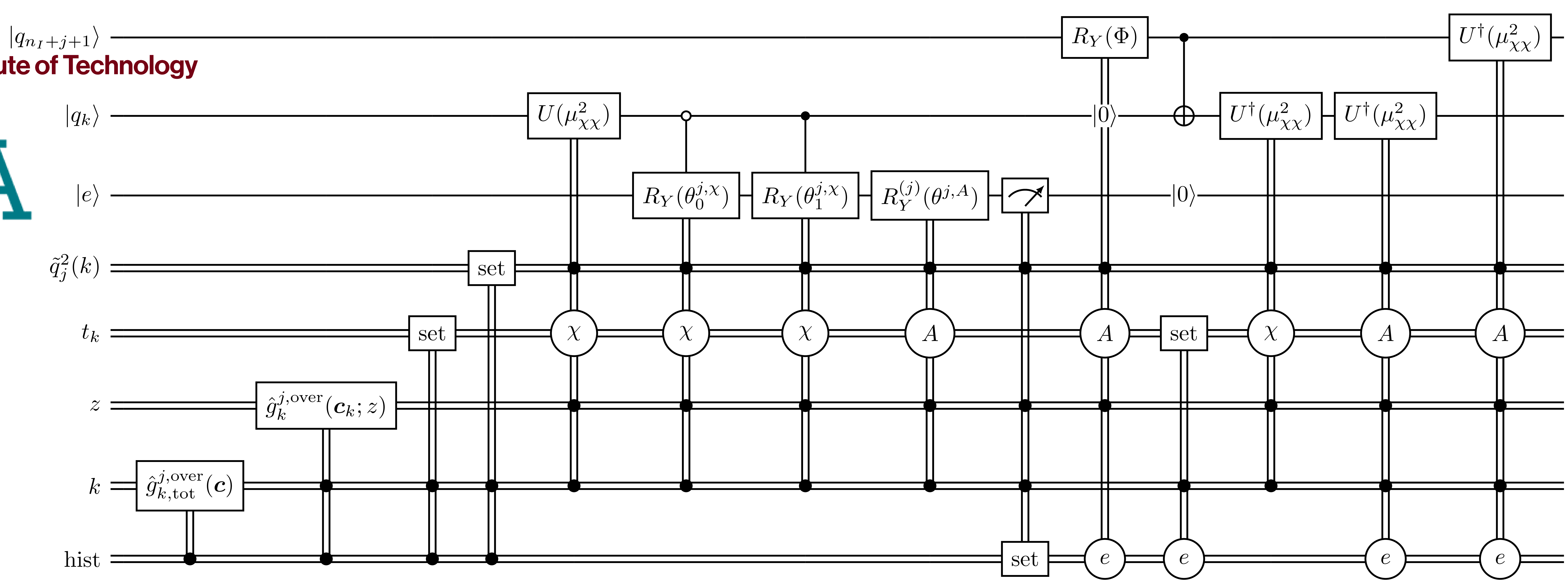
C²QA



Quantum simulation of parton shower with kinematics

In collaboration with C. W. Bauer and M. Yamazaki

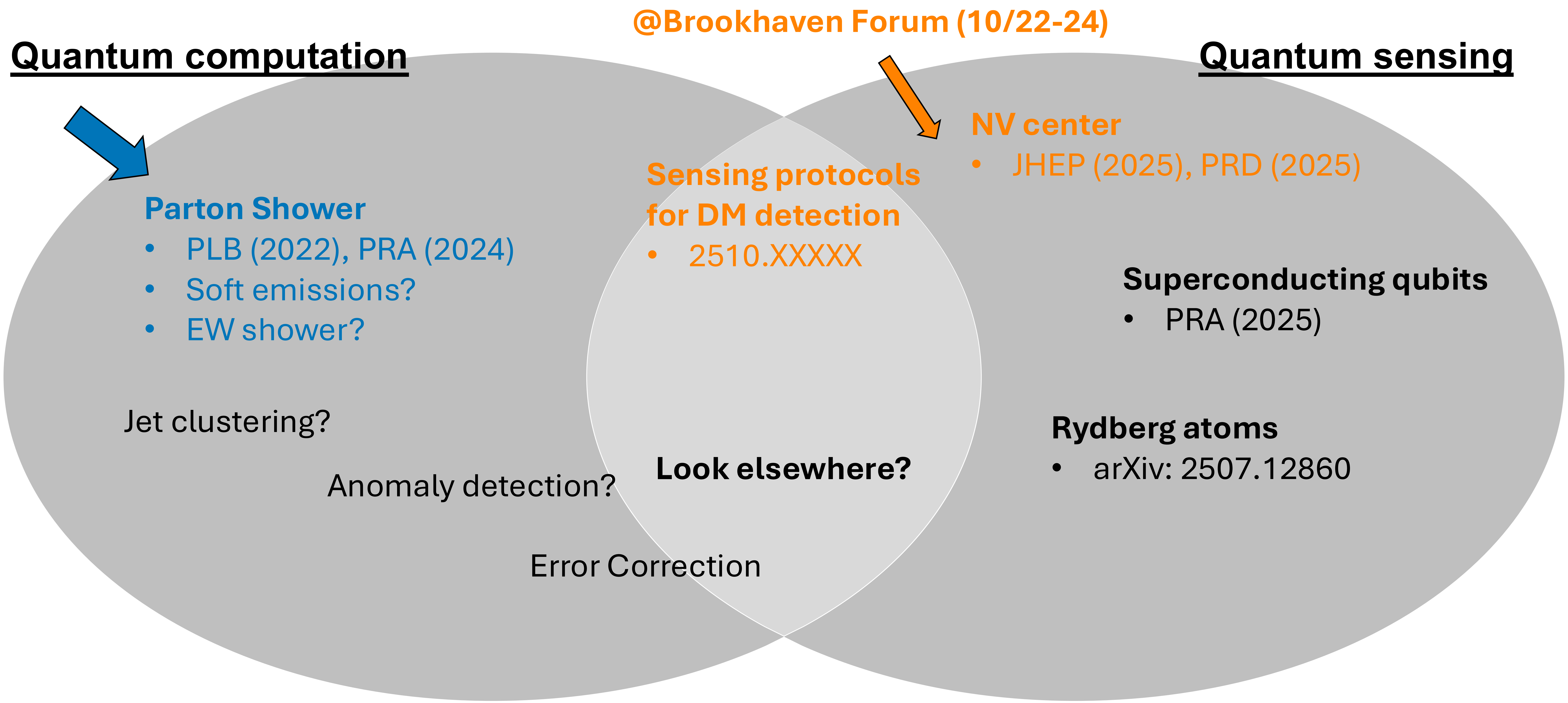
PLB 834 (2022) 137466 [arXiv: 2204.12500]

PRA 109 (2024) 3, 032432 [arXiv: 2310.19881]

So Chigusa

NT/RBRC Seminar @ BNL (10/17/2025)

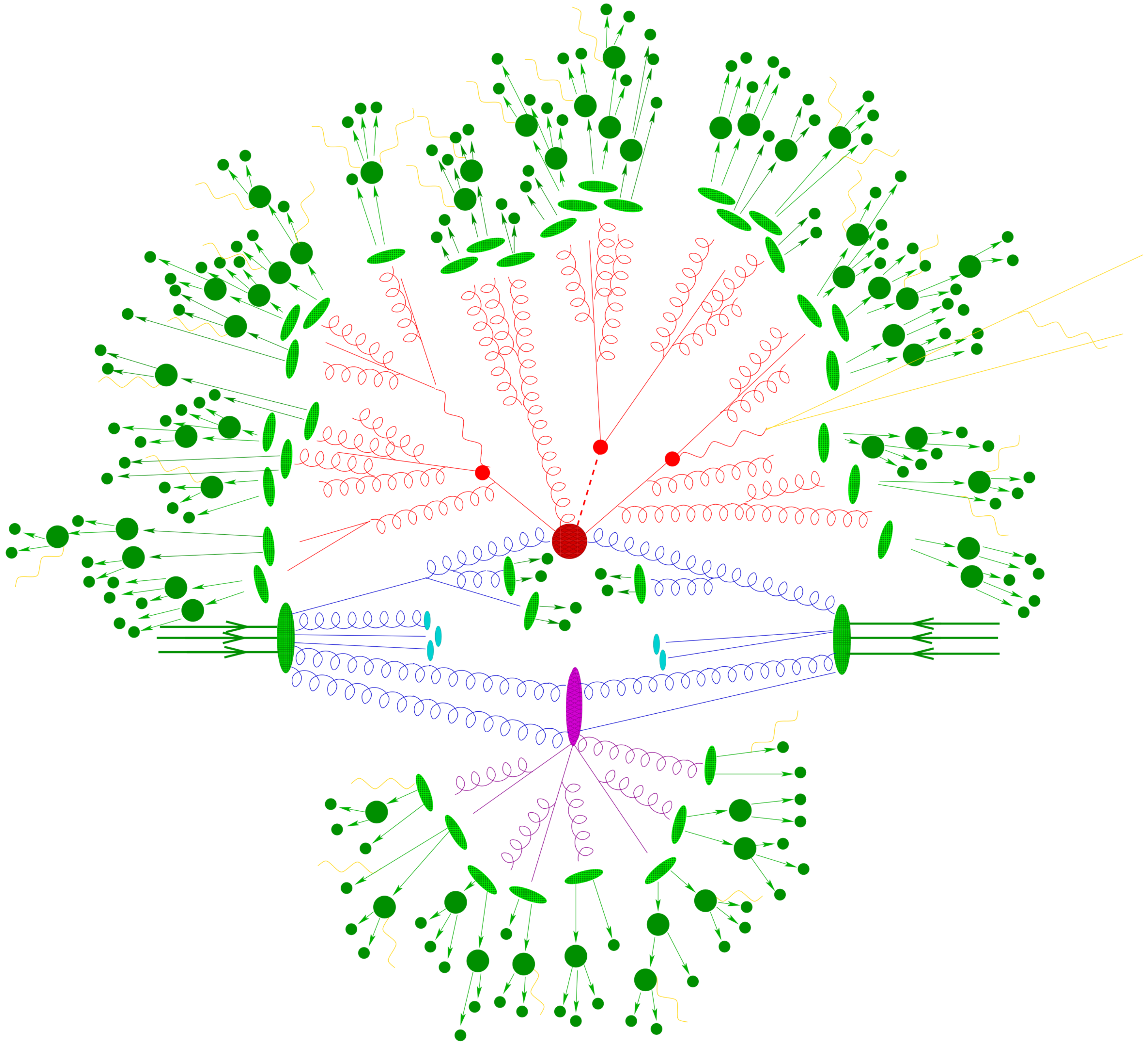
High-energy physics x Quantum science: my perspective



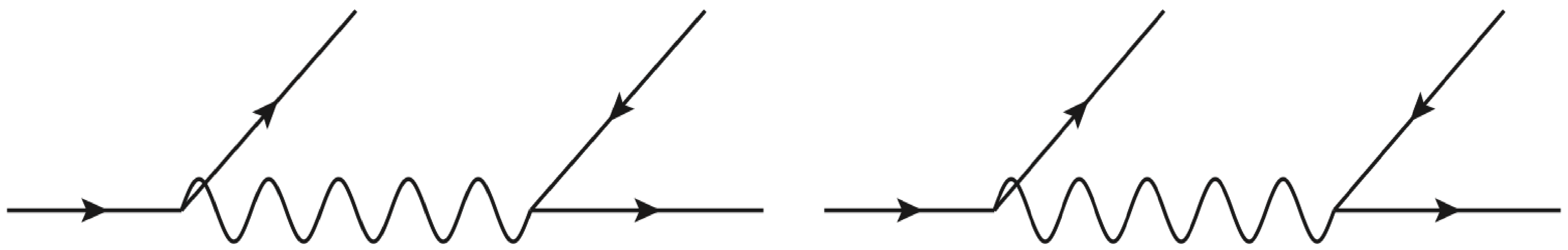
❖ Interesting interplay between computation and sensing motivated by high-energy physics!

Overview

- The known fact
  - Parton shower is a traditional algorithm to simulate high-energy multi-emission processes based on a classical probability distribution
- Problem
  - A non-trivial "flavor" structure could induce quantum interference effects, which cannot be tracked by the classical parton shower algorithm
- What we did
  - Constructed a quantum algorithm (QVPS) for simulation of kinematics
  - Demonstrated physics implications



Höche "Introduction to parton-shower event generators"



Z_T/γ Z_L/h

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A brief review of (classical) parton shower

- How it works
- When quantum interference become important
- Phenomenological implications of interference

Quantum Veto Parton Shower (QVPS) algorithm

C. W. Bauer, et al. [1904.03196]
P. Deliyannis, et al. [2203.10018] → Bauer, **SC**, Yamazaki '24

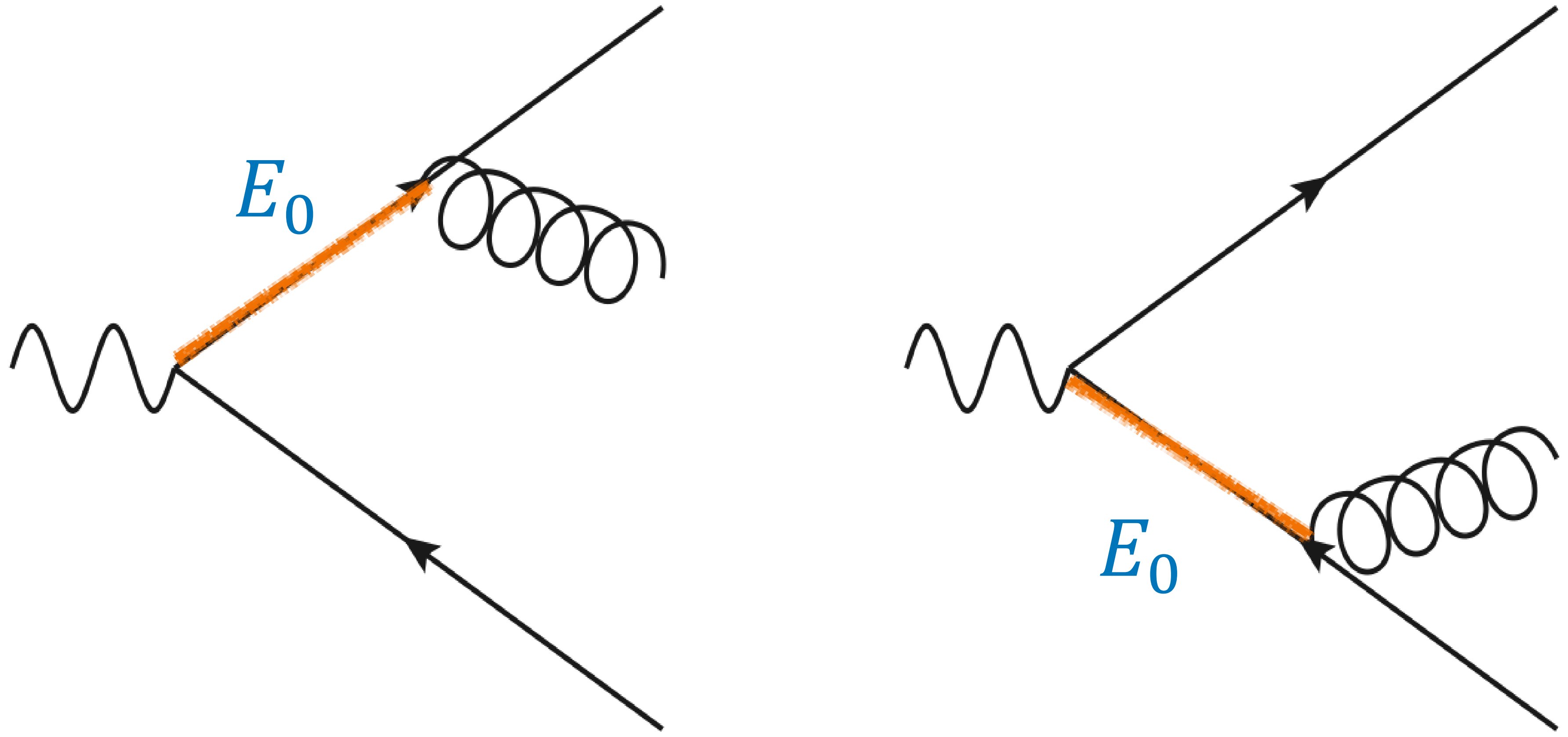
- Bottom-up demonstration of construction ideas
- How to incorporate kinematic information

Future directions

- Color interference
- Spin interference
- etc.

Large logarithms

- Soft/collinear singularities lead to an enhancement of emission processes
  - Ex) $q\bar{q} + g$ production



$$\sigma_{q\bar{q}g} \propto \sigma_{q\bar{q}} \frac{\alpha_s}{2\pi} \ln\left(\frac{E_0^2}{\mu_{\mathrm{IR}}^2}\right) \ln\left(\frac{E_0^2}{\mu_{\mathrm{IR}}^2}\right)$$

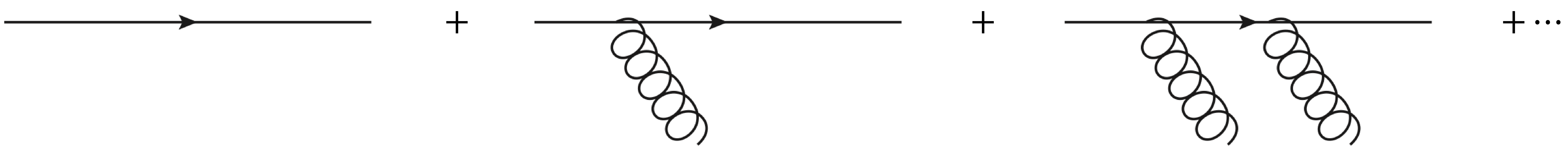
soft collinear

- The expansion parameter becomes larger - $\alpha \to \alpha \ln$

Resummation of large logarithms

- Emissions are not necessarily suppressions at high energy scales
  - Collinear emission @ LHC: $\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{E_0^2}{\Lambda_{QCD}^2}\right) \sim 30\% \;\Leftrightarrow\; E_0 \sim 0.6\mathrm{TeV}$
  - Soft & collinear γ @ muon collider: $\frac{\alpha}{2\pi}\ln^2\left(\frac{E_0^2}{m_\mu^2}\right) \sim 30\% \;\Leftrightarrow\; E_0 \sim 1\mathrm{TeV}$
  - Collinear emission from heavy DM: $\frac{\alpha_2(M_Z)}{2\pi}\ln\left(\frac{E_0^2}{m_Z^2}\right) \sim 30\% \;\Leftrightarrow\; E_0 \sim 0.5\mathrm{EeV}$

C. W. Bauer, et al. [2007.15001]

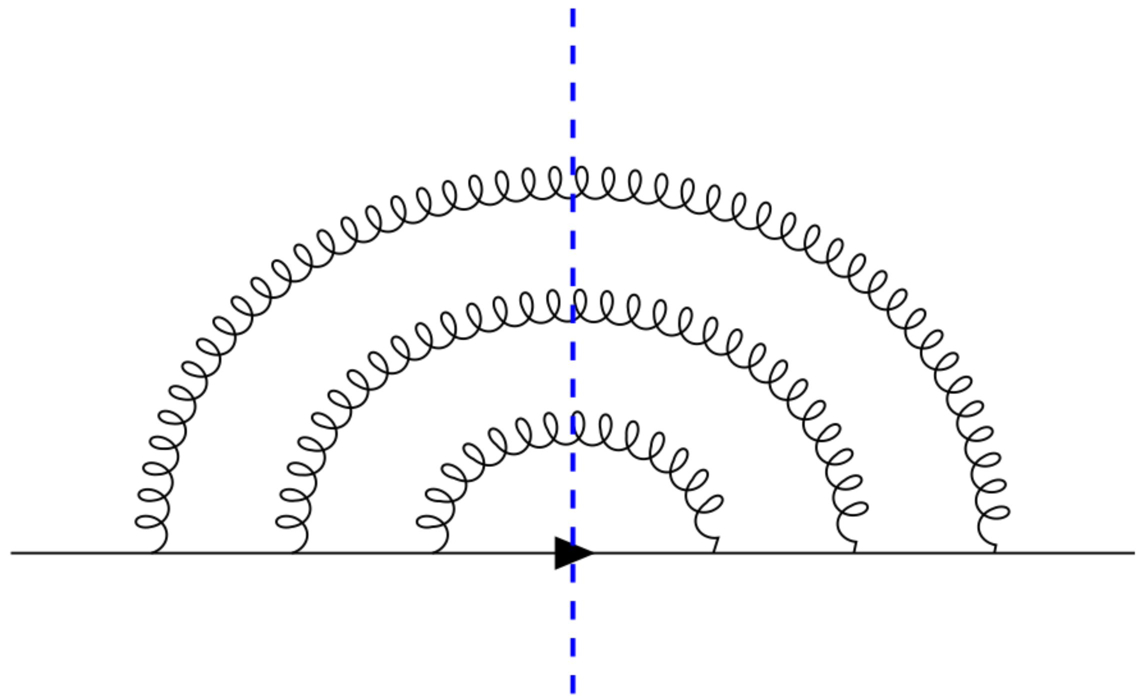


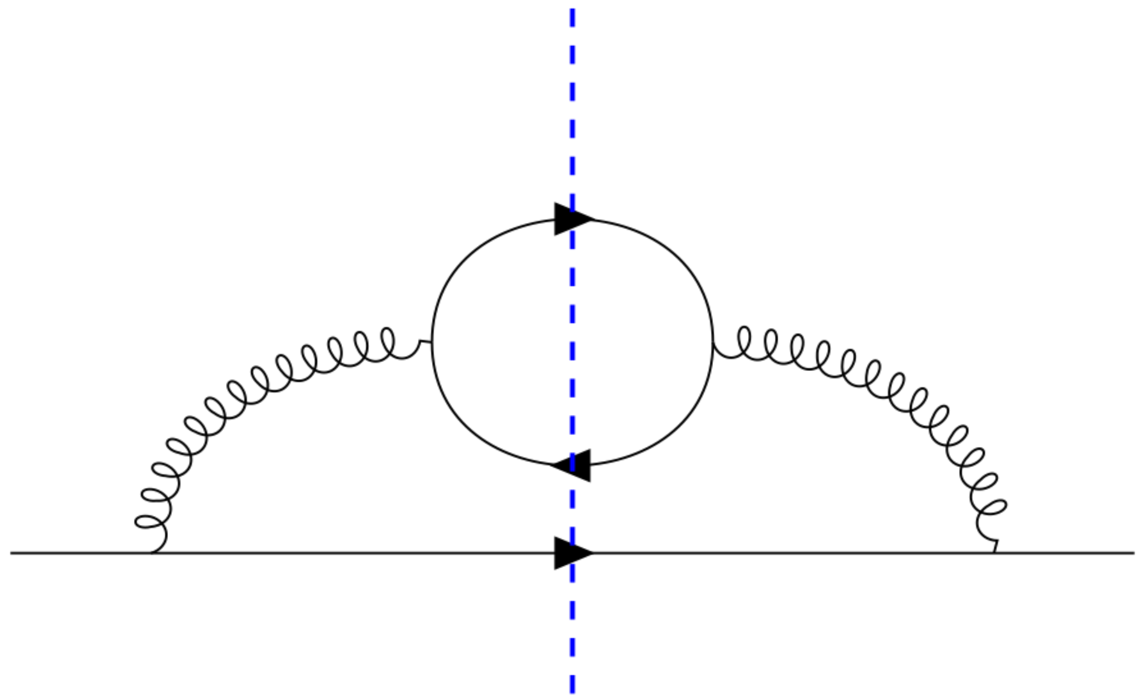
- Resummation of large logs needed!
  - The (collinear) leading logarithms (LL), $\sim \sum_n \left(\alpha \ln^{(\mathrm{collinear})}\right)^n$

Coherence

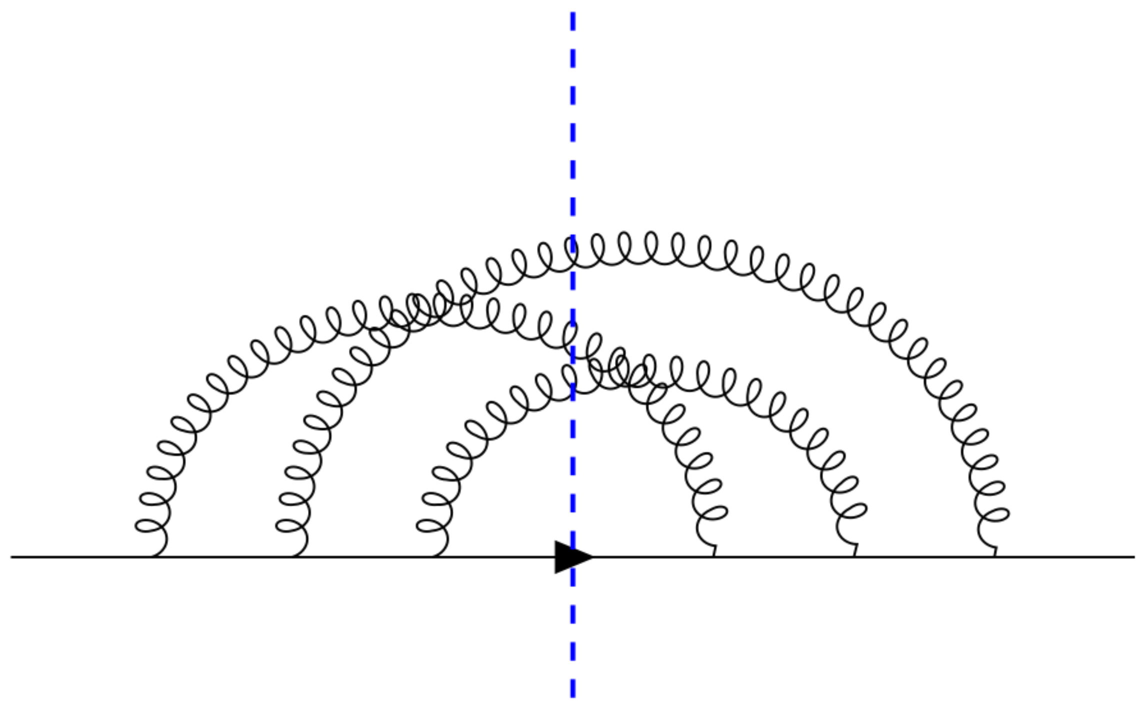
Only ladder-type diagrams with 1 → 2 splittings contribute at the collinear LL

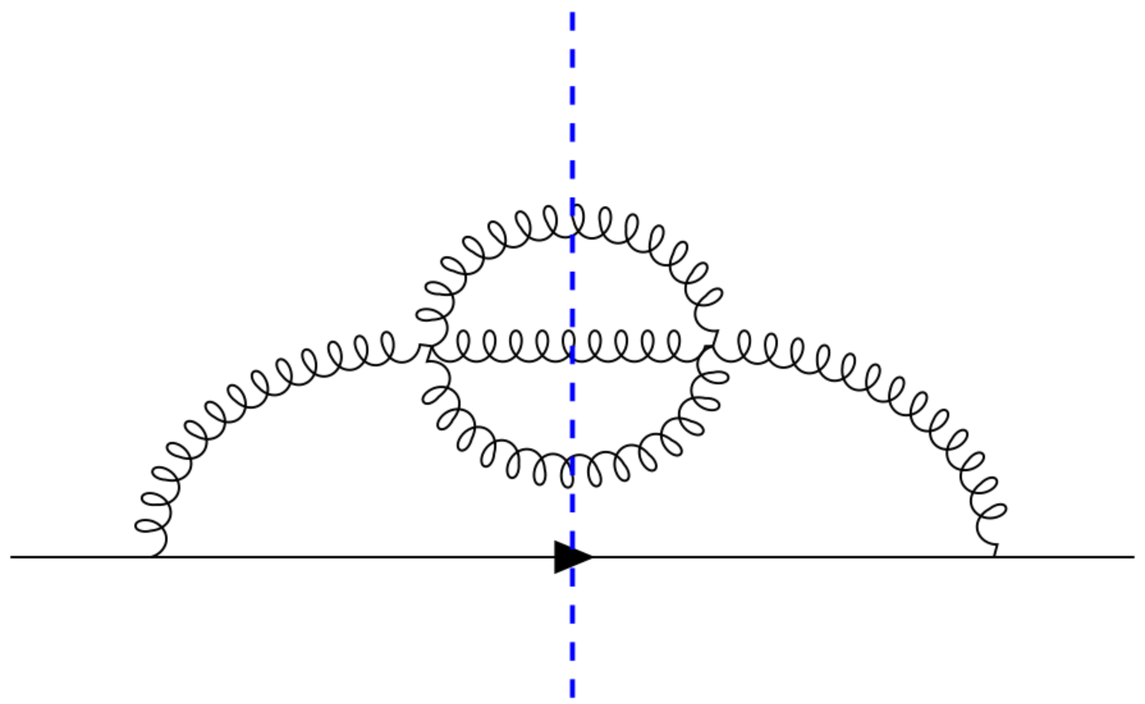
- ❖ LL contributions





- ❖ Beyond LL



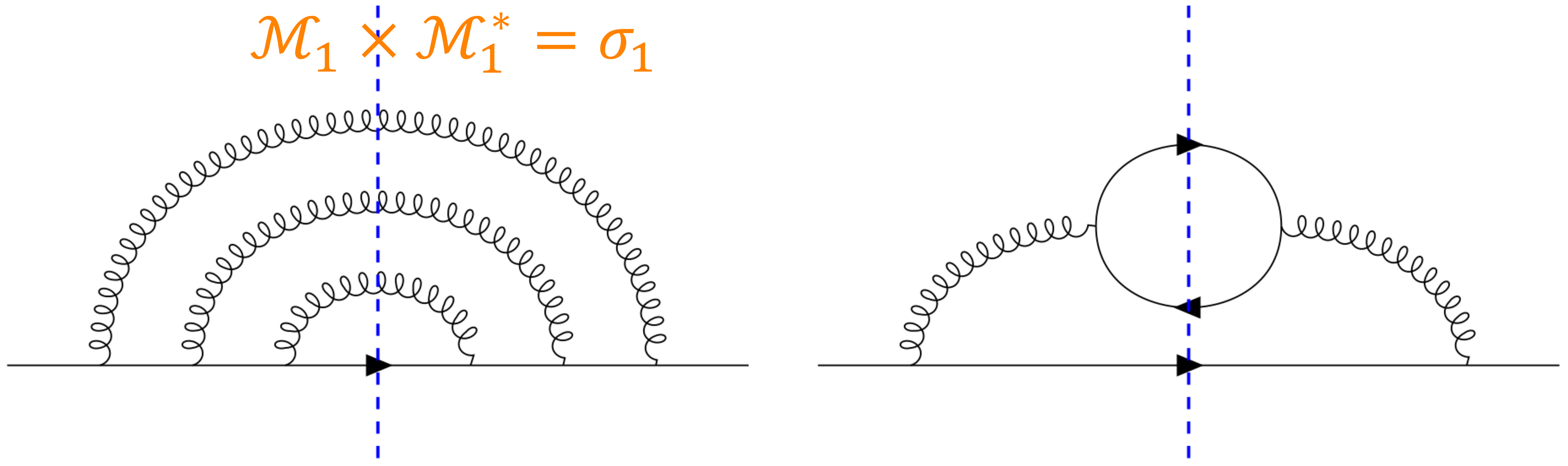


Coherence

Only ladder-type diagrams with $1 \to 2$ splittings contribute at the collinear LL

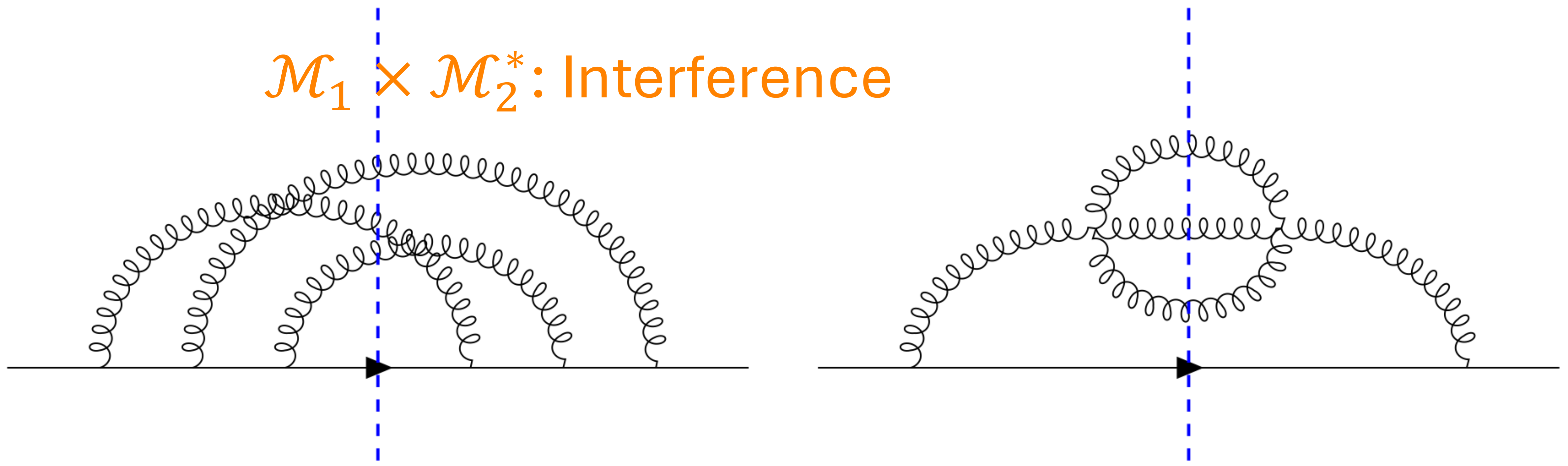
- LL contributions

$\mathcal{M}_1 \times \mathcal{M}_1^* = \sigma_1$



- Beyond LL

$\mathcal{M}_1 \times \mathcal{M}_2^*$: Interference



Chang[+] '70, Gribov[+] '72, Dokshitzer '77

Coherence

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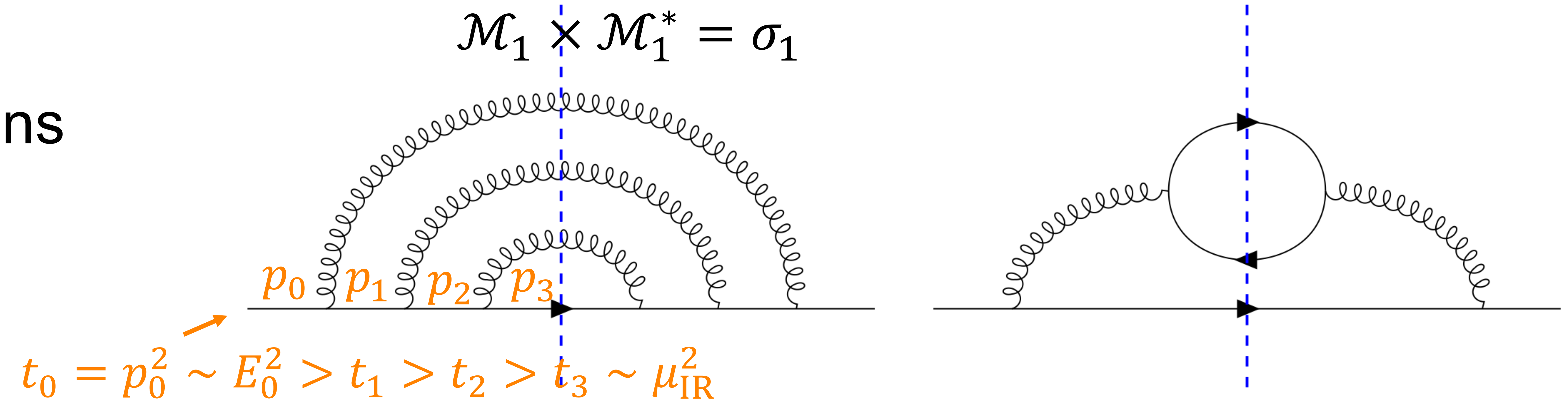
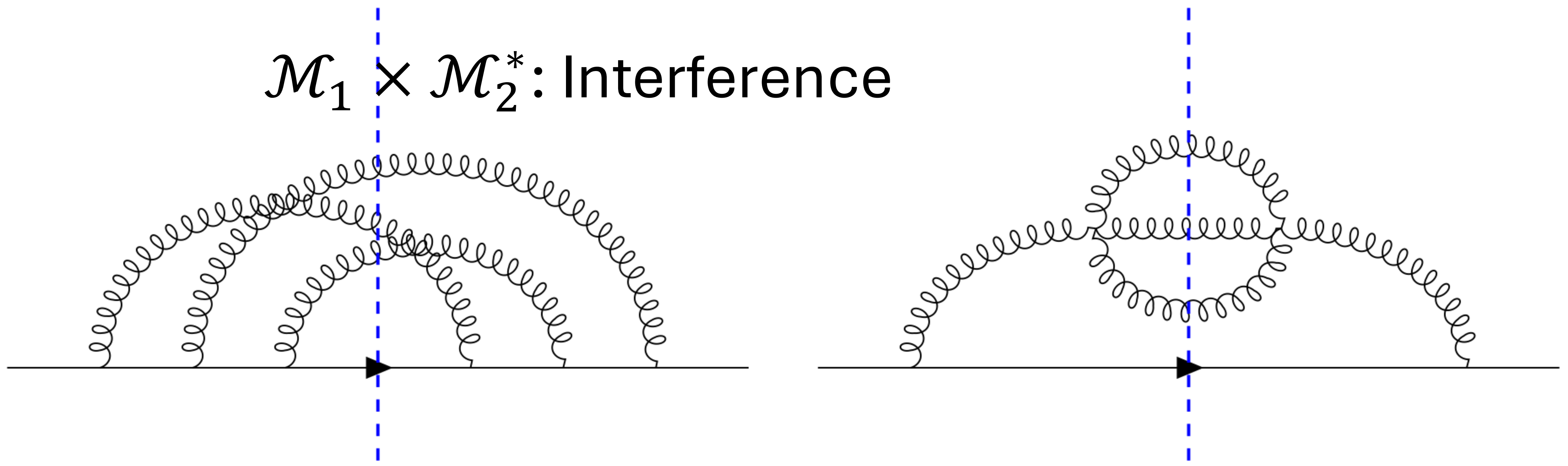
❖ LL contributions

$\mathcal{M}_1 \times \mathcal{M}_1^* = \sigma_1$



❖ Beyond LL

$\mathcal{M}_1 \times \mathcal{M}_2^*$: Interference

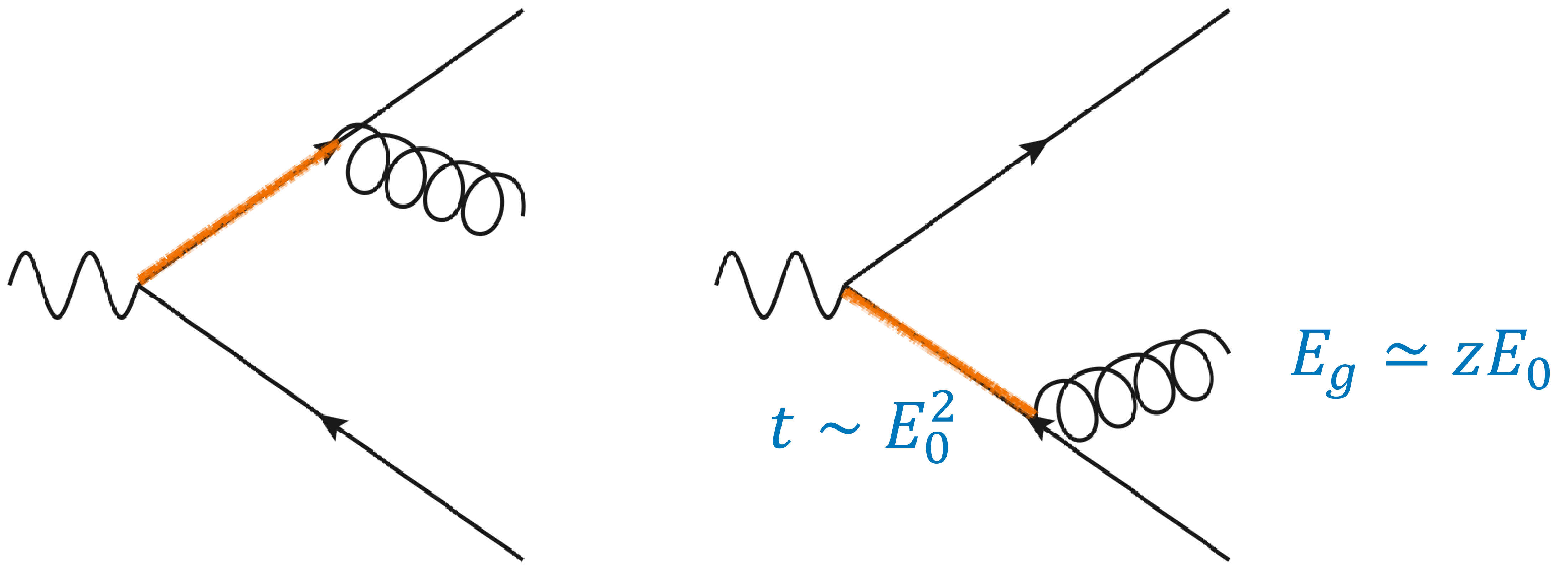


- Virtuality ordering is another requirement

Chang[+] '70, Gribov[+] '72, Dokshitzer '77

Cross-section relations

- The relationship among cross sections
  - Ex) $q\bar{q} + g$ production



$$\frac{d\sigma_{q\bar{q}g}}{dtdz} \simeq \sigma_{q\bar{q}} \sum_{q,\bar{q}} \frac{\alpha_s}{2\pi} \frac{1}{t} C_F \frac{1+(1-z)^2}{z}$$

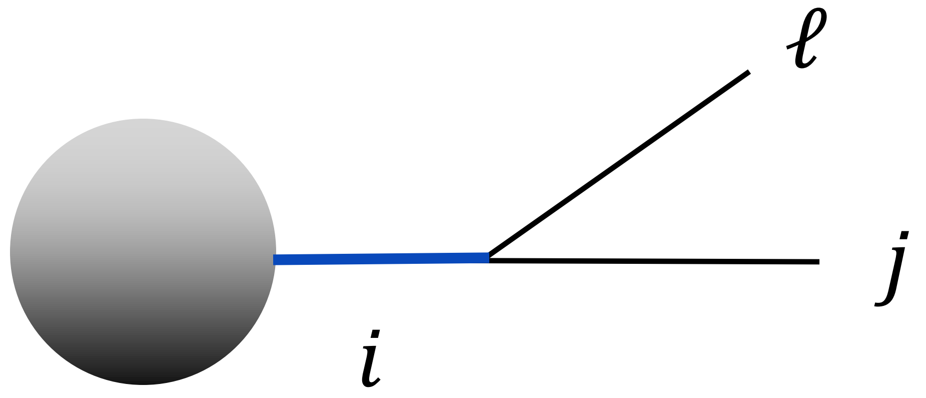
$$\begin{cases} t \in \left[\Lambda_{QCD}^2, E_0^2\right]: \text{Virtuality} \\ z \in [0,1] \simeq \text{Energy fraction} \end{cases}$$

- Can be interpreted as classical “splitting probabilities”

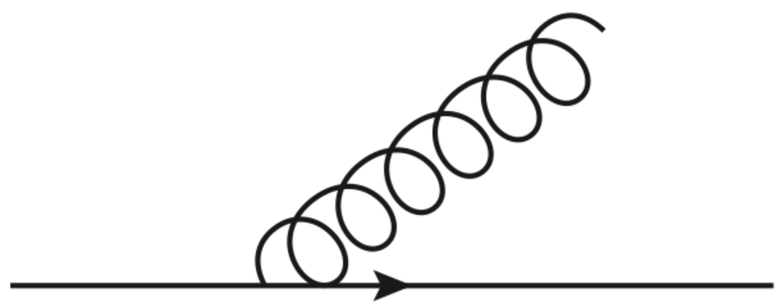
$$d\mathcal{P}_{q\to gq} = d\mathcal{P}_{\bar{q}\to g\bar{q}} \simeq \frac{\alpha_s}{2\pi} \frac{dt}{t} C_F \frac{1+(1-z)^2}{z} dz$$

General splitting and splitting functions

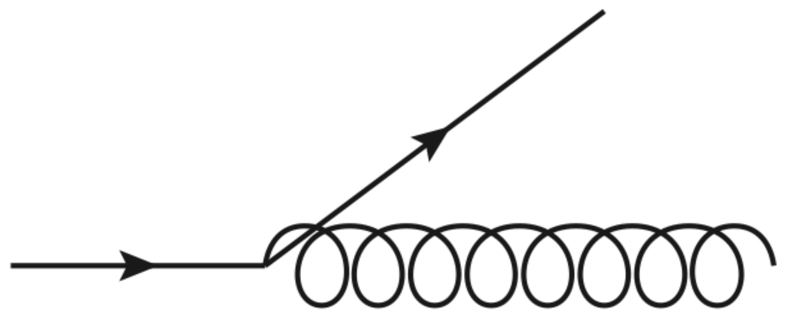
- Factorization is general ⇨ general splitting probability
  - $d\mathcal{P}_{i\to j\ell} \simeq \frac{\alpha(t,z)}{2\pi}\frac{dt}{t}P_{i\to j\ell}(z)dz$



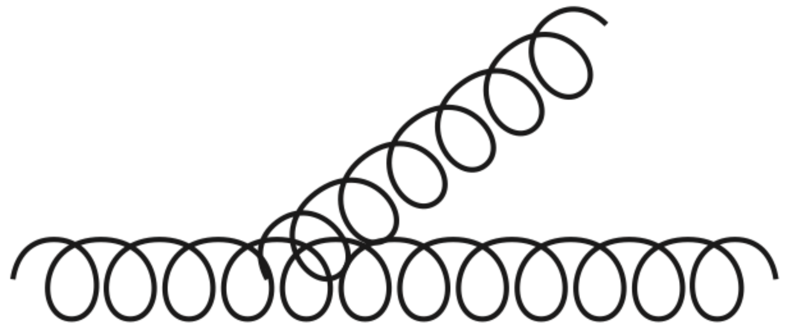
- Splitting functions in QCD



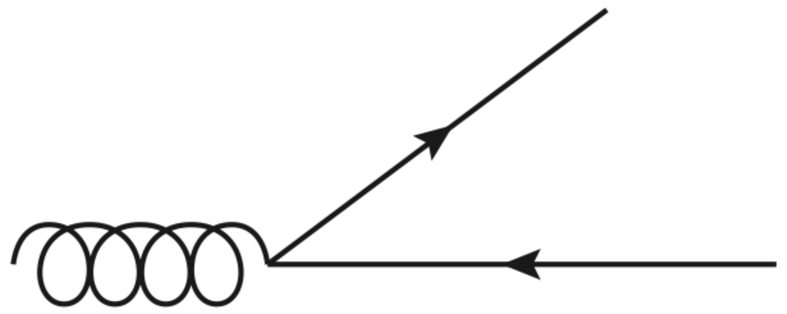
$$P_{q\to qg} = C_F\frac{1+z^2}{1-z}$$



$$P_{q\to gq} = C_F\frac{1+(1-z)^2}{z}$$



$$P_{g\to gg} = 2C_A\frac{(1-z(1-z))^2}{z(1-z)}$$



$$P_{g\to q\bar{q}} = T_R z^2(1-z)^2$$

Classical parton shower

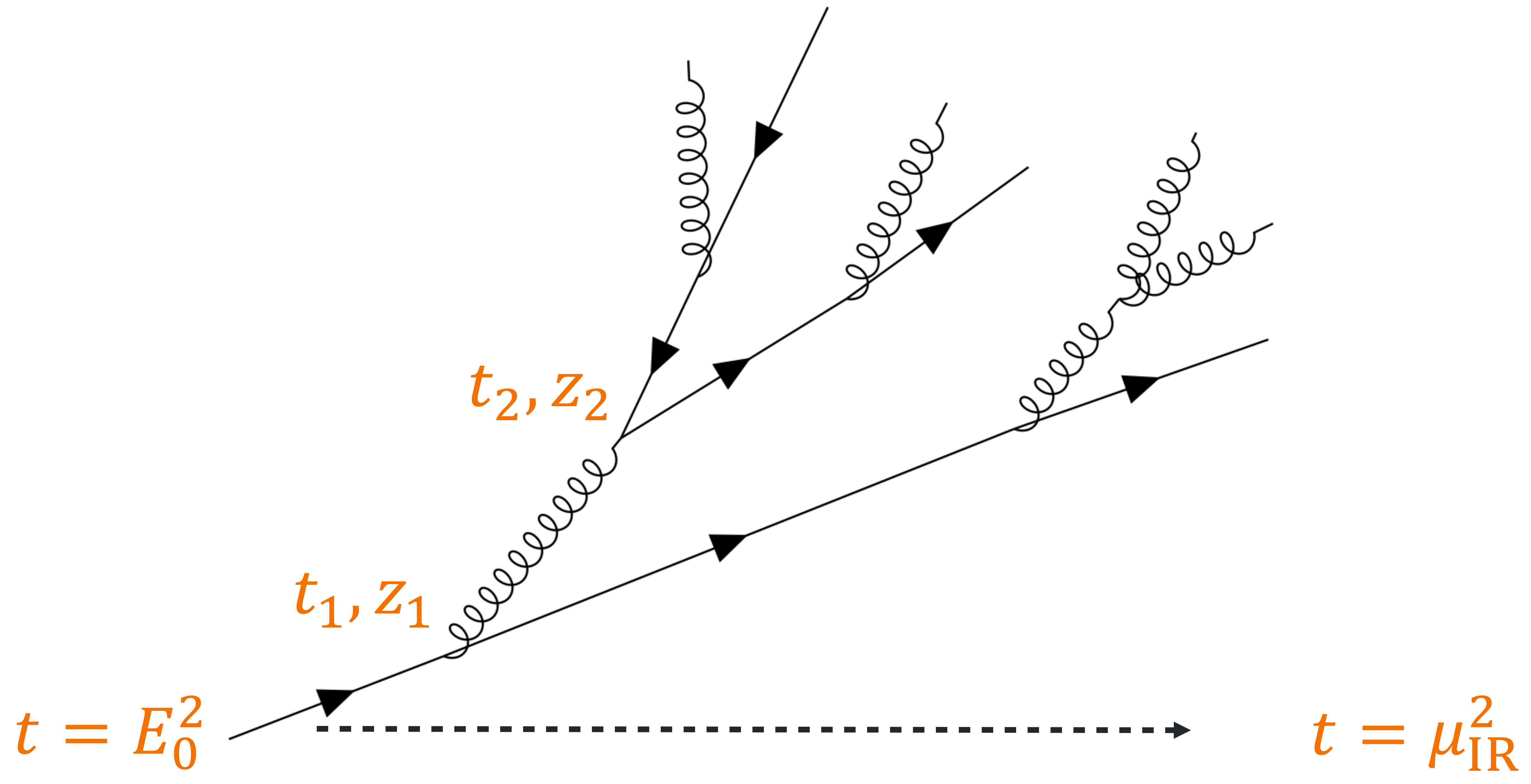
❖ Is a Monte Carlo simulation that simulates multi-emission processes with

$$d\mathcal{P}_{i\to j\ell} \simeq \frac{\alpha(t,z)}{2\pi}\frac{dt}{t}P_{i\to j\ell}(z)dz$$

❖ Some well-known public codes

- Pythia8
- Herwig
- Sherpa

cf) Angular ordering Marchesini+ '84, '88



❖ Large log resummation is reshuffling of cross sections (ensured by unitarity)

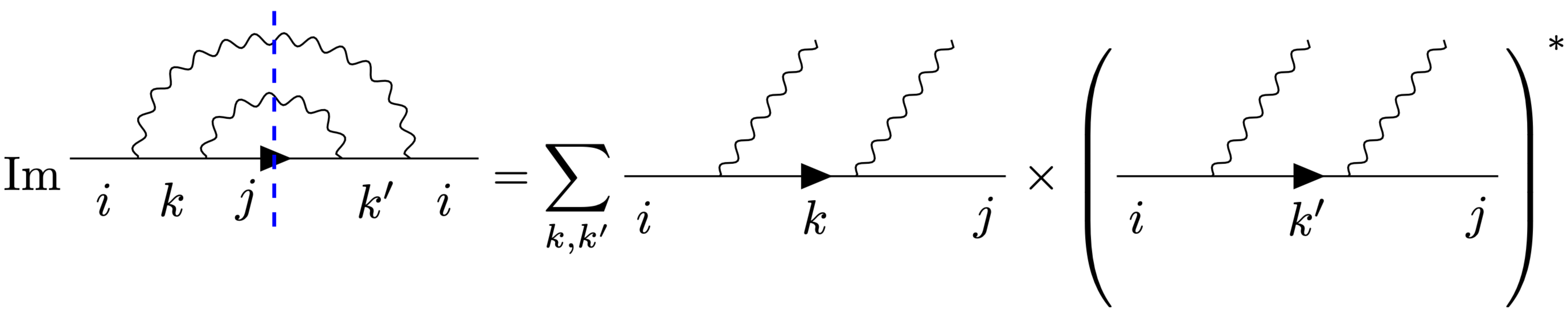
- e.g., $\sigma_{q\bar{q}}^{\mathrm{LO}} = \sigma_{q\bar{q}}^{\mathrm{LO+LL}} + \sigma_{q\bar{q}g}^{\mathrm{LO+LL}} + \sigma_{q\bar{q}gg}^{\mathrm{LO+LL}} + \sigma_{q\bar{q}q\bar{q}}^{\mathrm{LO+LL}} + \cdots$

Quantum interference in parton shower

❖ A loophole in the discussion so far

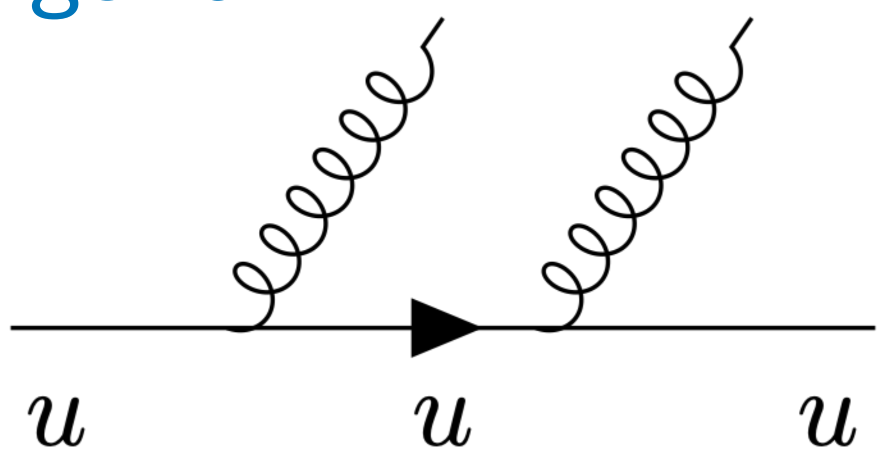
A non-trivial flavor structure makes interference effects important at the LL-level

$\mathcal{M}_1 \times \mathcal{M}_2^*$: Interference

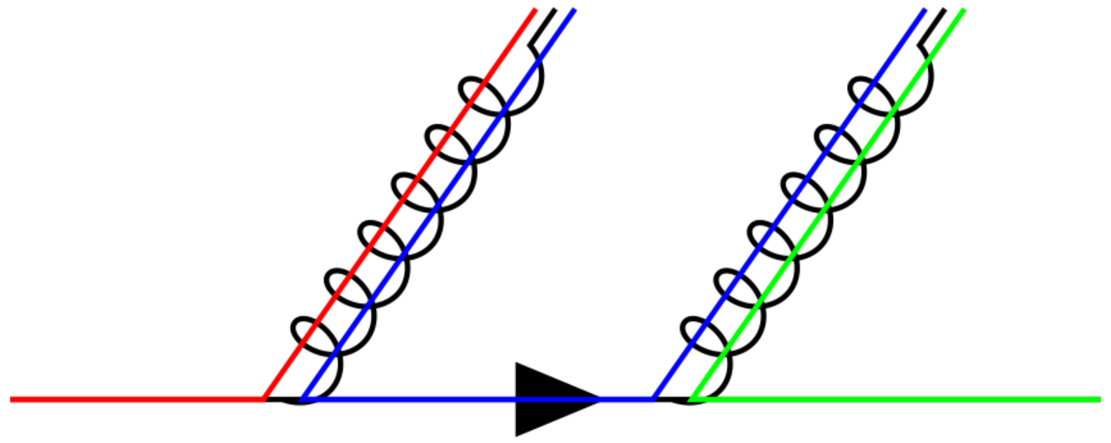


❖ QCD was OK (@ inclusive LO simulations)

- Flavor diagonal



- Color is classical information @ LO of N_c

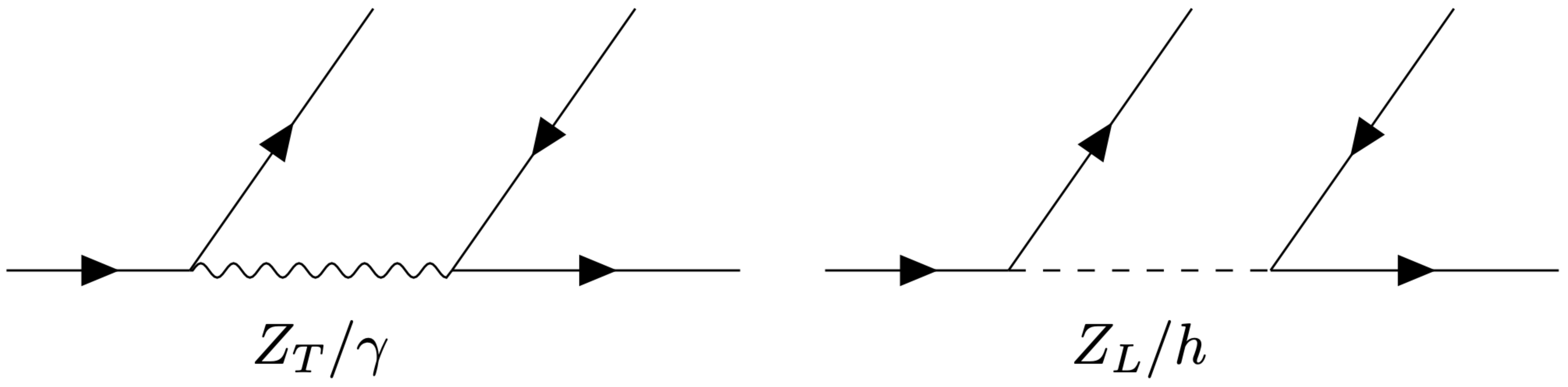


Models with quantum interference

- EW shower
  - Classical treatment

    Z. Nagy, E. Soper [0706.0017]

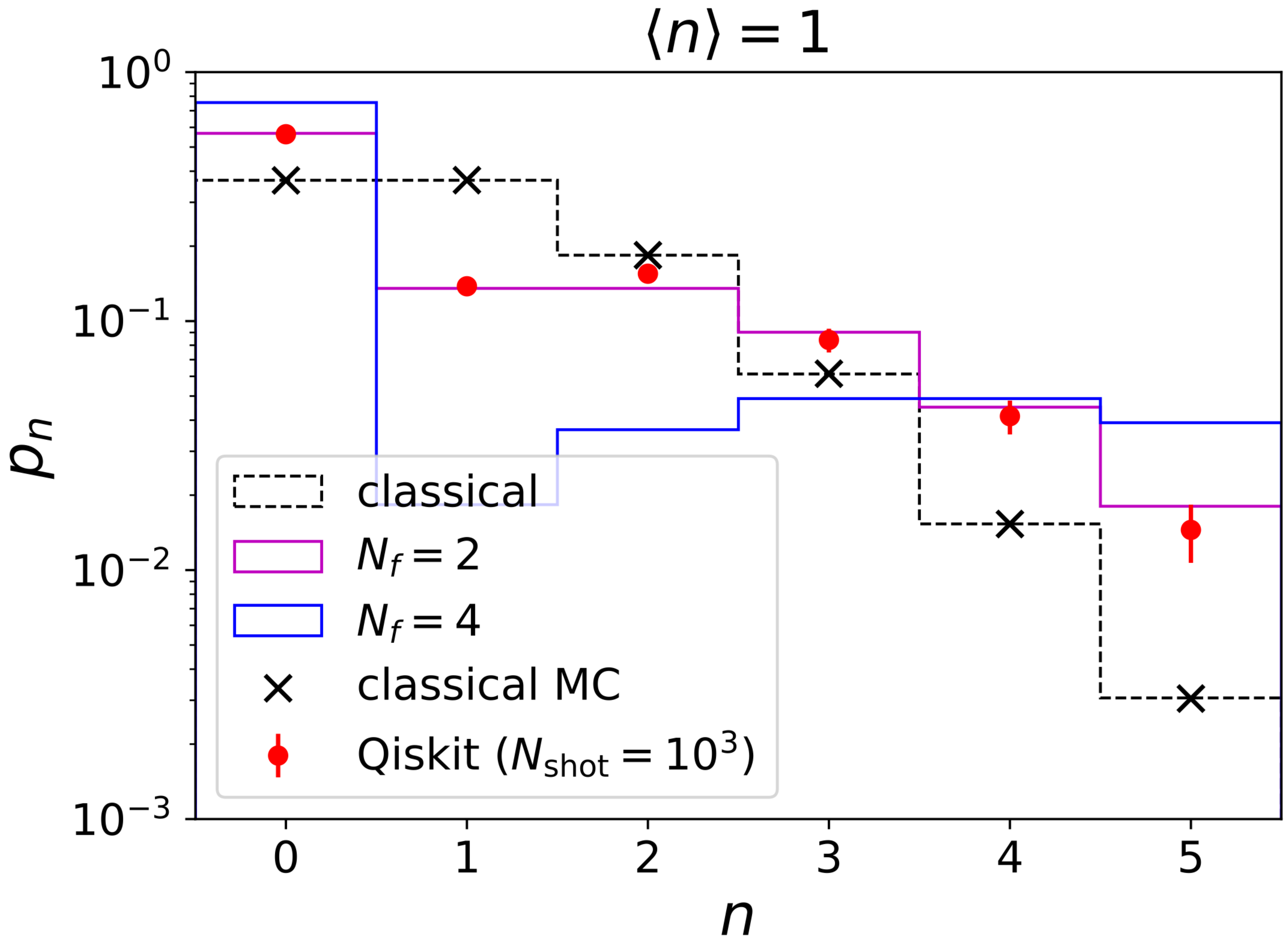
    J. Chen, T. Han, B. Tweedie [1611.00788]



- Simple toy model: N_f fermions charged under dark $U(1)$
  - $\mathcal{L}_{\text{dark}} = \sum_i \bar{\chi}_i (i\partial - m_\chi)\chi_i + \sum_{i,j} i g_{ij} \bar{\chi}_i A' \chi_j - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} - \frac{1}{2} m_{A'}^2 A'_\mu A'^\mu$

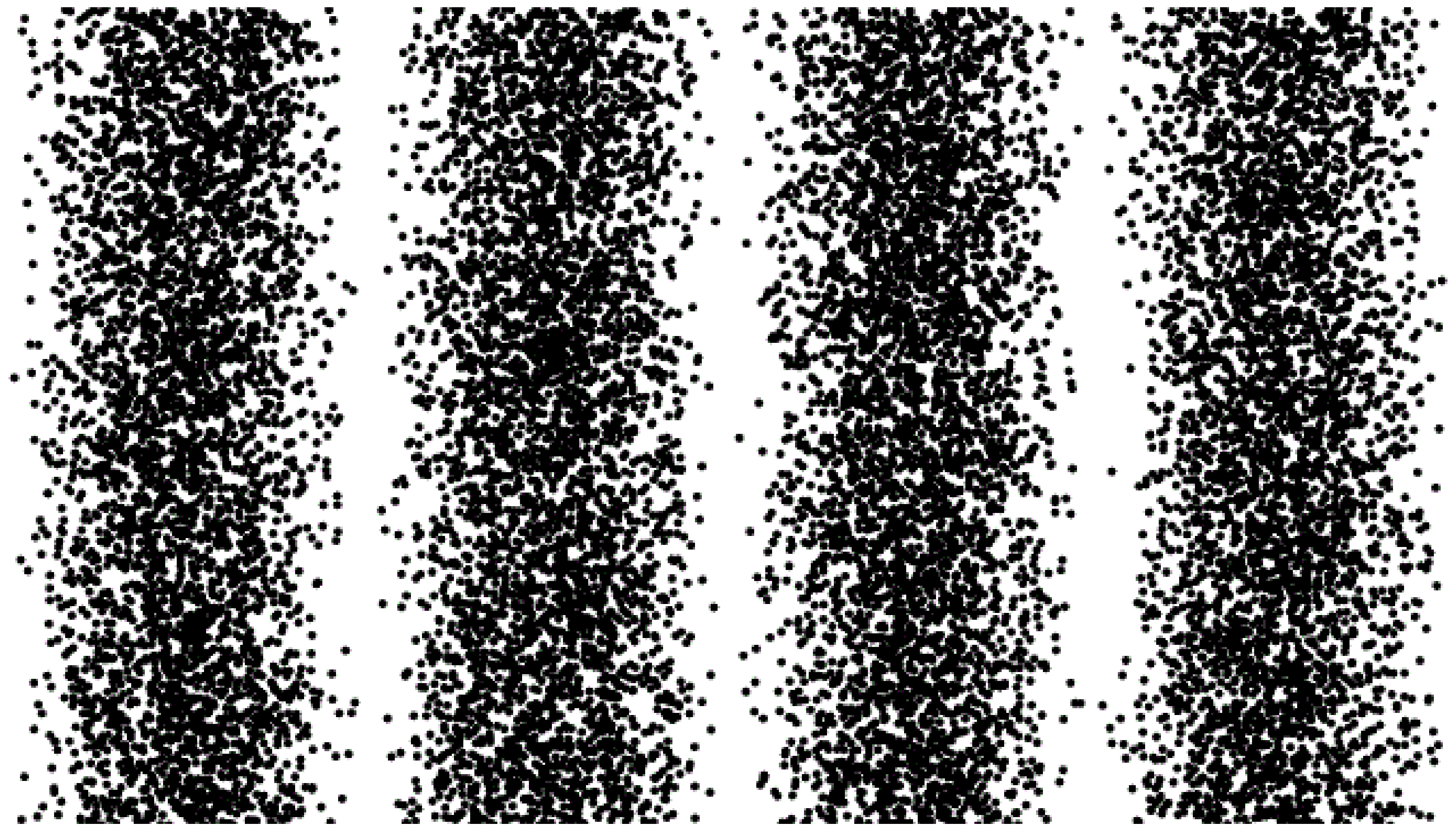
- Classical parton shower simulation can not take account of quantum interference effects
  - Possible phenomenological impact

Distribution of the number of emissions



SC, Yamazaki '22

From classical to quantum simulation

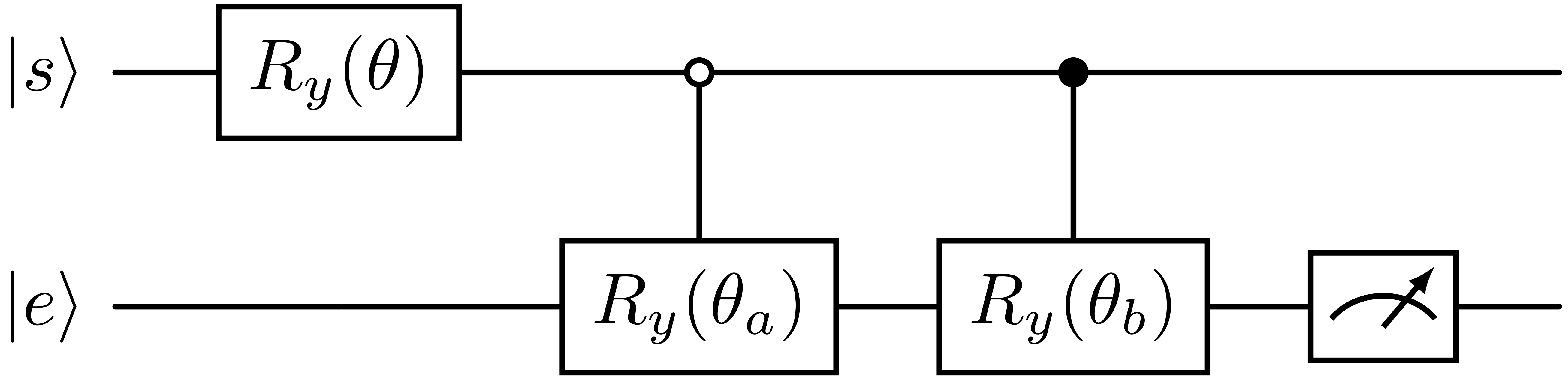


- The interference effect is a fundamental feature of the quantum mechanics
- Can we naturally take account of this by quantum simulation?
  - “Amplitude-level” solution: store flavor information as a superposition of quantum states!

Simplest two-flavor example

C. W. Bauer, et al. [1904.03196]

$t \in [t_{j+1}, t_j]$



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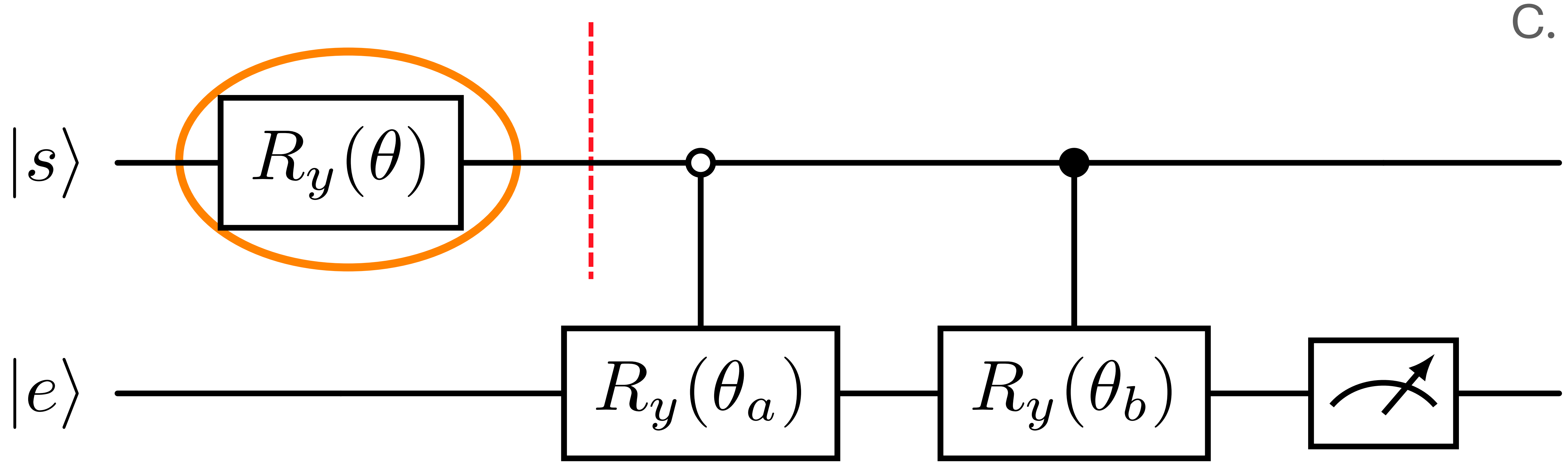


- $|s\rangle$ stores flavor information of a parton

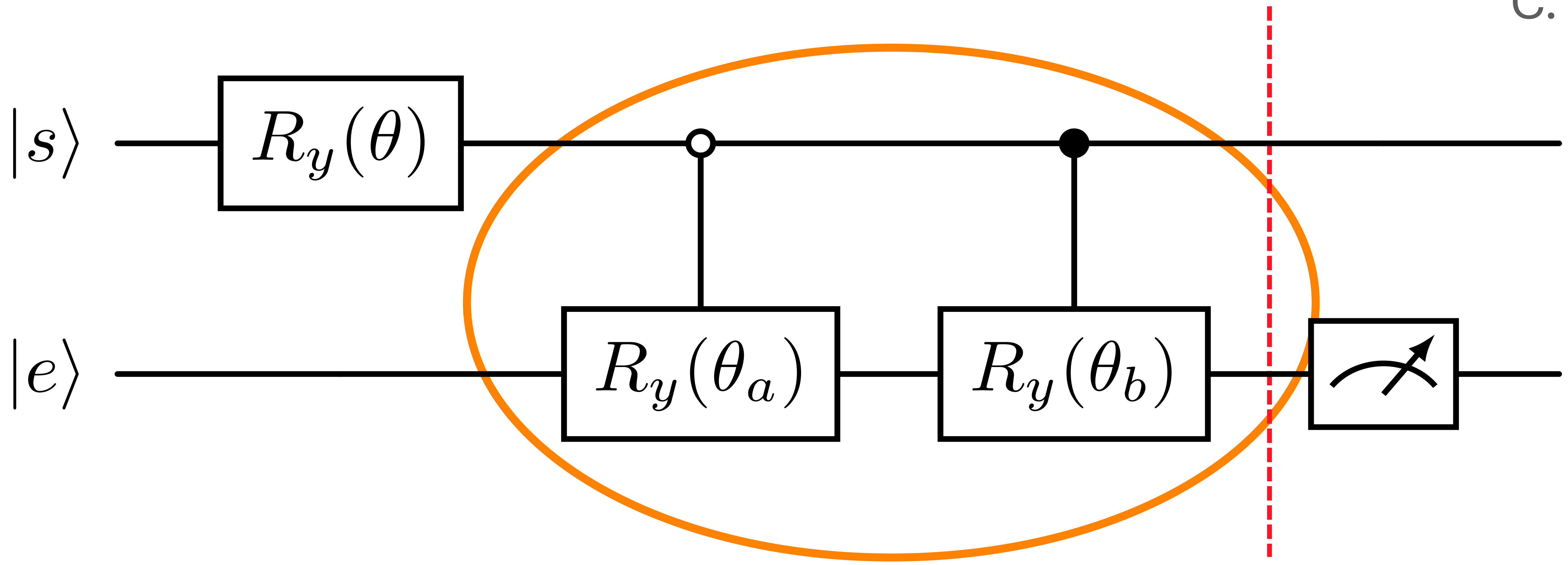
$$|s\rangle = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \cos\frac{\theta}{2}|a\rangle + \sin\frac{\theta}{2}|b\rangle$$

- Flavor basis to interaction basis: $\mathcal{L}_{\text{int}} = iA' \begin{pmatrix} \bar{\chi}_1 \\ \bar{\chi}_2 \end{pmatrix} \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} (\chi_1 \quad \chi_2) = \tilde{g}_a \bar{\chi}_a A' \chi_a + \tilde{g}_b \bar{\chi}_b A' \chi_b$

Simplest two-flavor example

C. W. Bauer, et al. [1904.03196]

$t \in [t_{j+1}, t_j]$



- $|e\rangle$ preserves whether the emission occurs or not

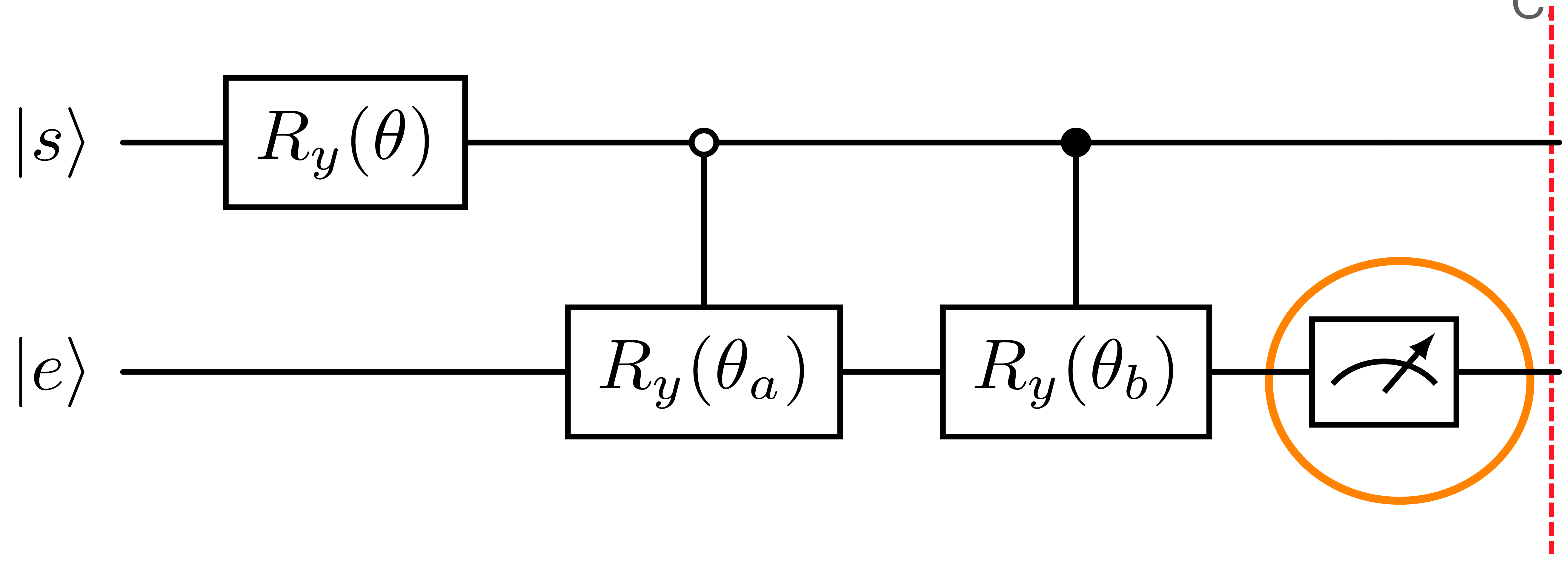
$$\cos\frac{\theta}{2}|a\rangle\left(\cos\frac{\theta_a}{2}|0_e\rangle + \sin\frac{\theta_a}{2}|1_e\rangle\right) + \sin\frac{\theta}{2}|b\rangle\left(\cos\frac{\theta_b}{2}|0_e\rangle + \sin\frac{\theta_b}{2}|1_e\rangle\right)$$

- Emission probability from $|q\rangle$ $(q = a, b)$ - $p_q = \sin^2\frac{\theta_q}{2}$

Simplest two-flavor example

C. W. Bauer, et al. [1904.03196]

$t \in [t_{j+1}, t_j]$



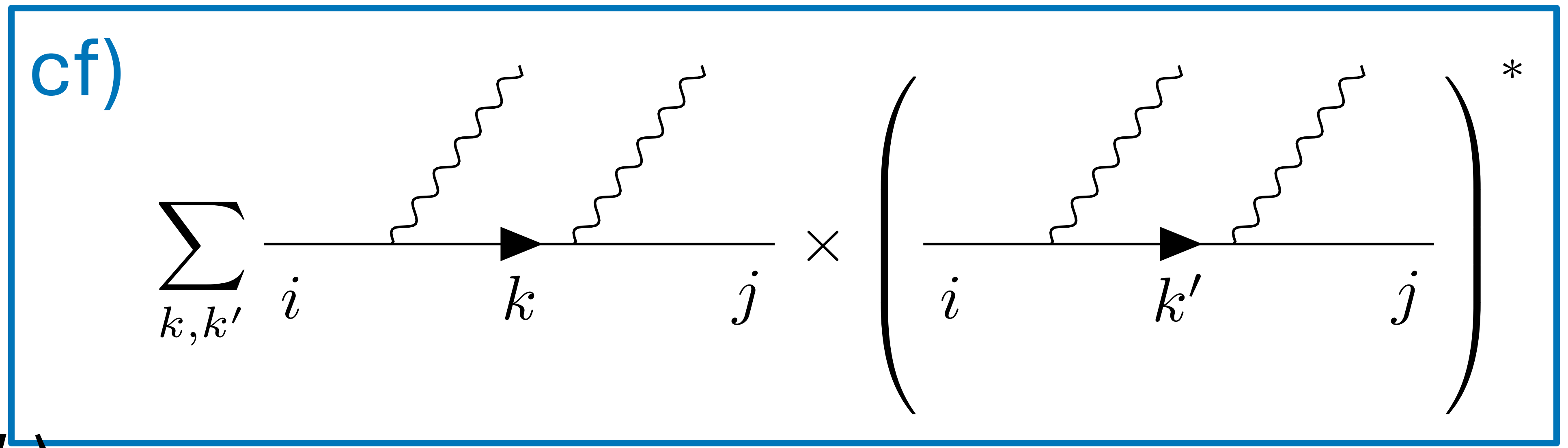
❖ Measurement affects both the $|s\rangle$ and $|e\rangle$ states

$$\cos\frac{\theta}{2}|a\rangle\left(\cos\frac{\theta_a}{2}|0_e\rangle + \sin\frac{\theta_a}{2}|1_e\rangle\right) + \sin\frac{\theta}{2}|b\rangle\left(\cos\frac{\theta_b}{2}|0_e\rangle + \sin\frac{\theta_b}{2}|1_e\rangle\right) \quad \text{(before meas.)}$$

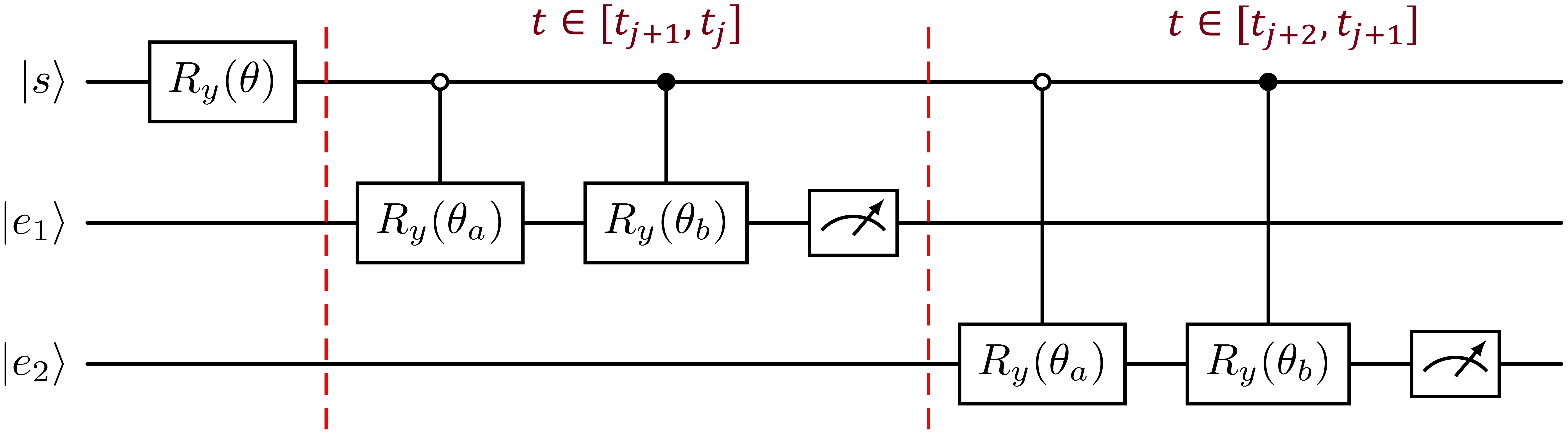
$$\Rightarrow |\psi\rangle \propto \left(\cos\frac{\theta}{2}\cos\frac{\theta_a}{2}|a\rangle + \sin\frac{\theta}{2}\cos\frac{\theta_b}{2}|b\rangle\right)|0_e\rangle \quad (e = 0)$$

$$\Rightarrow |\psi\rangle \propto \left(\cos\frac{\theta}{2}\sin\frac{\theta_a}{2}|a\rangle + \sin\frac{\theta}{2}\sin\frac{\theta_b}{2}|b\rangle\right)|1_e\rangle \quad (e = 1)$$

Quantum interference effect



❖ $(N=2)$-step simulation starting from $|s\rangle = c_{\theta/2}|a\rangle + s_{\theta/2}|b\rangle$



❖ “Classical” anticipation

- $p_{e=1}^{(N=1)} = c_{\theta/2}^2 \Delta\mathcal{P}_a + s_{\theta/2}^2 \Delta\mathcal{P}_b$
- $p_{e_1=e_2=1}^{(N=2)} = \left(p_{e=1}^{(N=1)}\right)^2$

❖ Quantum result

- $p_{e=1}^{(N=1)} = c_{\theta/2}^2 \Delta\mathcal{P}_a + s_{\theta/2}^2 \Delta\mathcal{P}_b$
- $p_{e_1=e_2=1}^{(N=2)} = c_{\theta/2}^2 \Delta\mathcal{P}_a^2 + s_{\theta/2}^2 \Delta\mathcal{P}_b^2 \neq \left(p_{e=1}^{(N=1)}\right)^2$

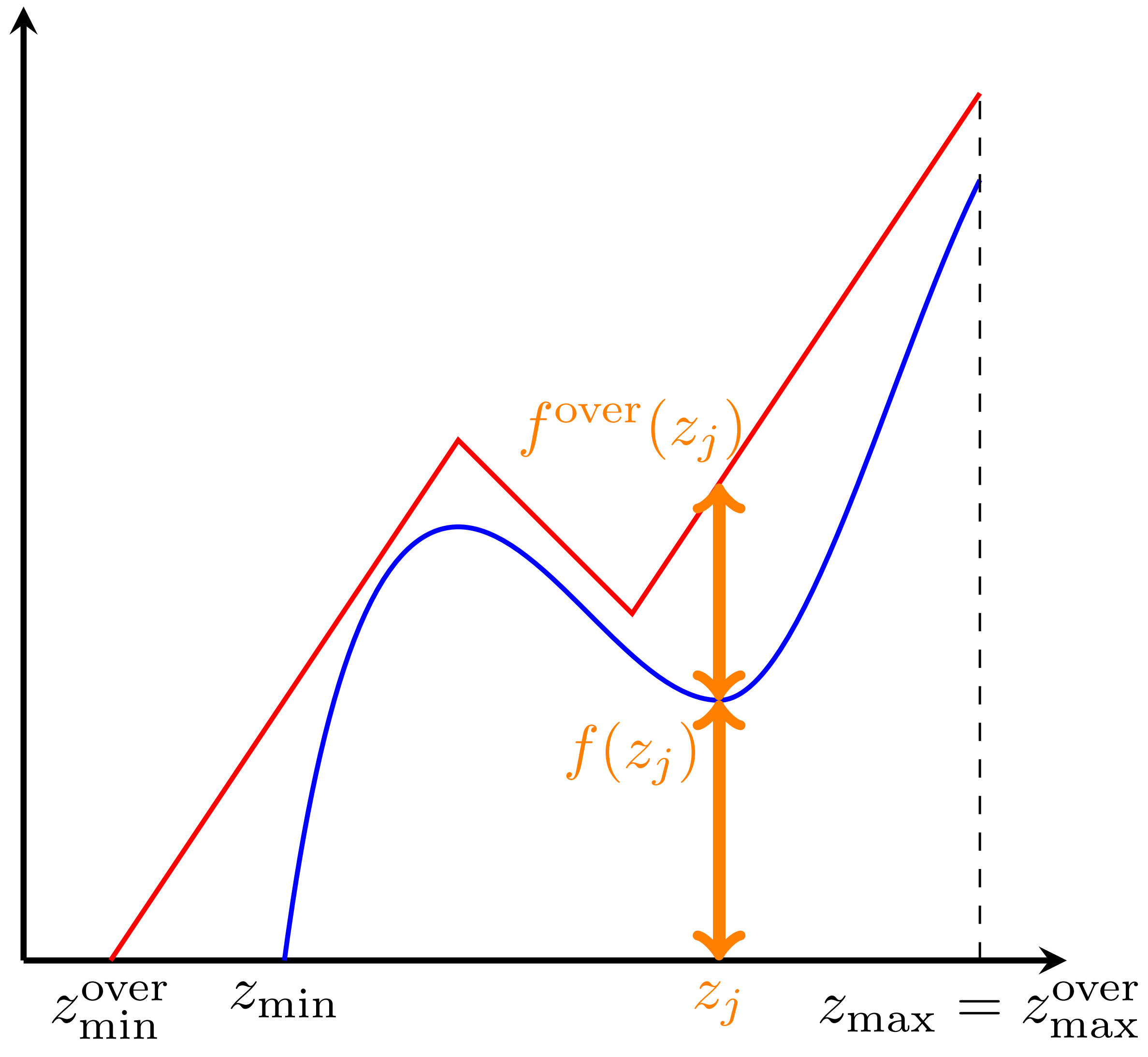
Towards sampling: veto method

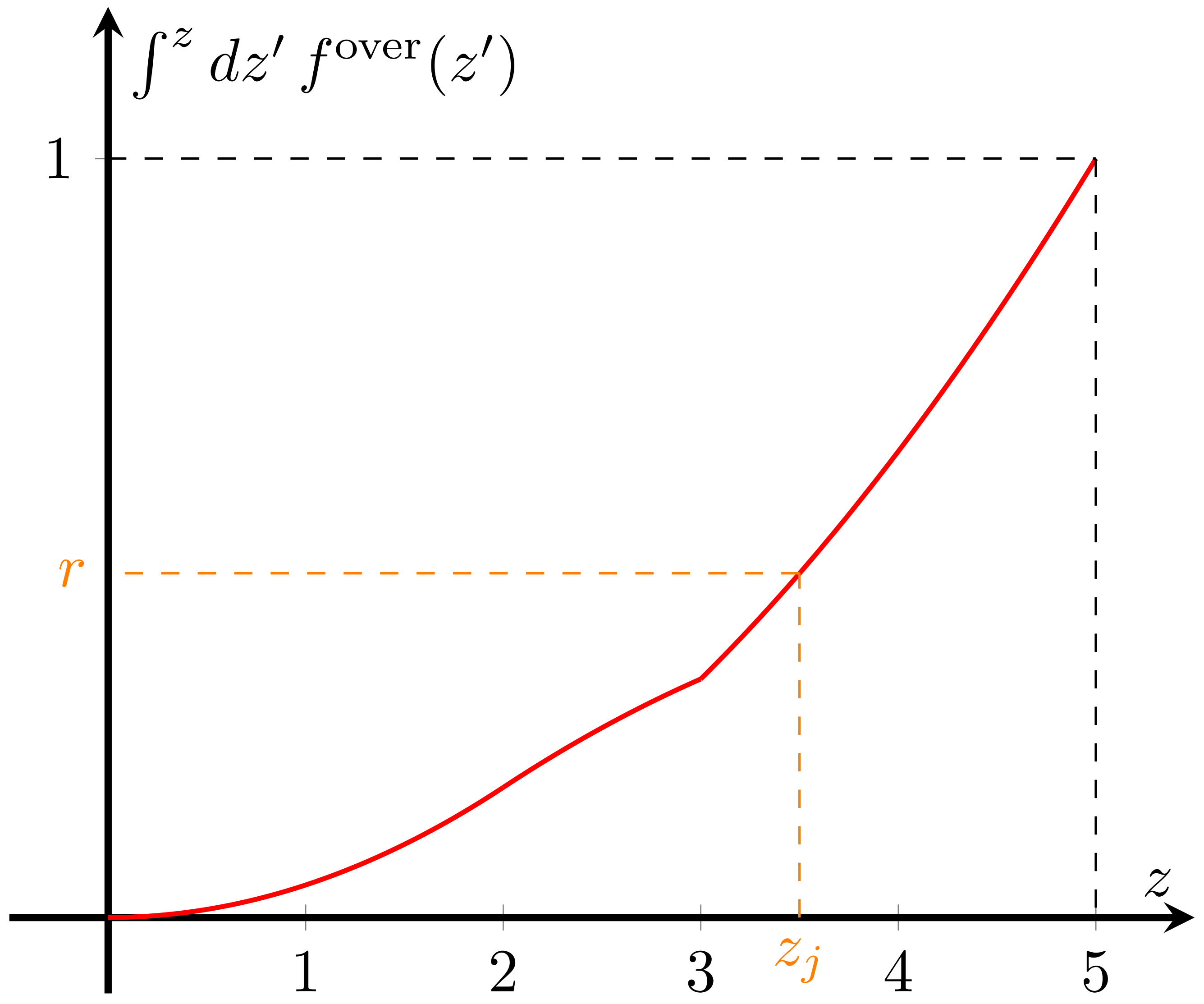
❖ We judge if emission occurs in $t \in [t_{j+1}, t_j]$ and sample z according to

$$\Delta\mathcal{P} \simeq \ln\frac{t_j}{t_{j+1}} \times \int_{z_{\min}(t_j)}^{z_{\max}(t_j)} dz\,\frac{\alpha(t_j, z)}{2\pi} P(z) \leq 1$$

❖ The veto method for sampling based on a complicated distribution $f(z)$

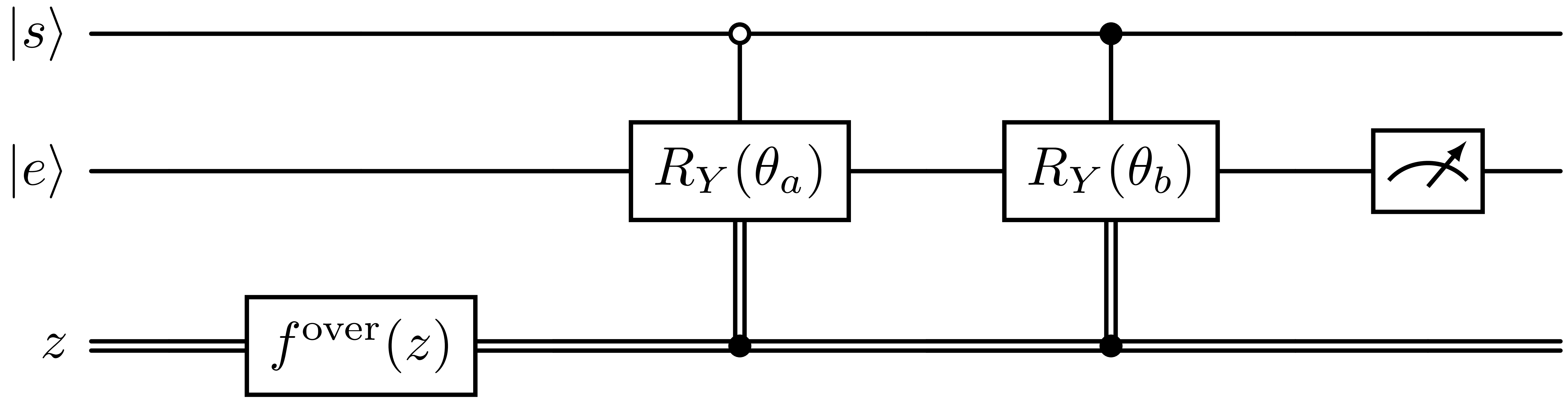
1) Prepare over-estimated quantities
   - $f^{\text{over}}(z) \geq f(z)$ with $\int_{z_{\min}^{\text{over}}}^{z_{\max}^{\text{over}}} dz\, f^{\text{over}}(z) = 1$
   - $[z_{\min}^{\text{over}}, z_{\max}^{\text{over}}] \supseteq [z_{\min}, z_{\max}]$
2) Sample z_j according to $f^{\text{over}}(z)$
   - Solve $\int_{z_{\min}^{\text{over}}}^{z_j} dz' f^{\text{over}}(z') = r \in [0,1)$
3) Veto (= conclude no emission) if
   - $z_j \notin [z_{\min}, z_{\max}]$ or
   - $f(z_j)/f^{\text{over}}(z_j) < r' \in [0,1)$





Two-flavor simulation with sampling

$t \in [t_{j+1}, t_j]$



Two-flavor simulation with sampling

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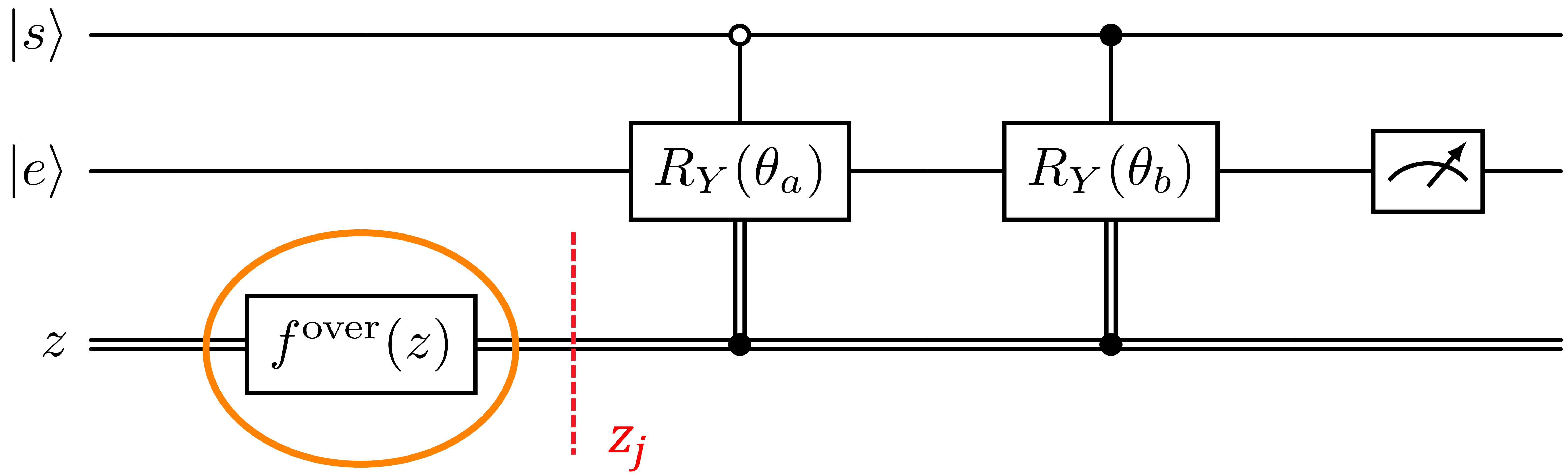
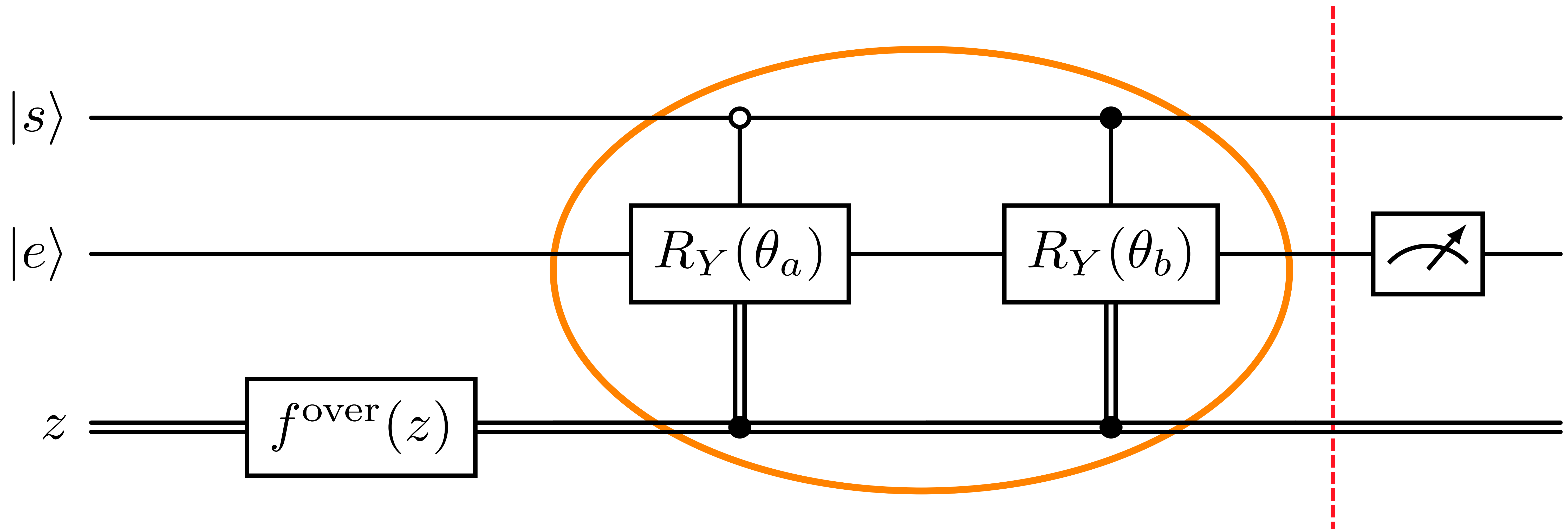


❖ Sample z according to over-estimated quantities with $f^{\text{over}}(z) \geq \max(f_a(z), f_b(z))$

Two-flavor simulation with sampling

$t \in [t_{j+1}, t_j]$

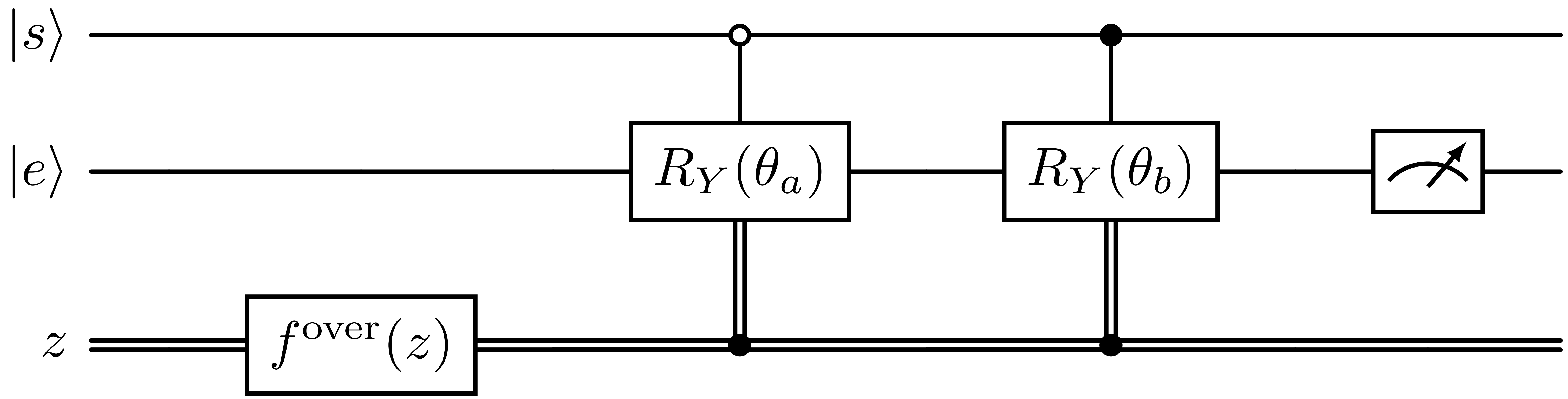


- Sample z according to over-estimated quantities with $f^{\text{over}}(z) \geq \max\left(f_a(z), f_b(z)\right)$

- State-dependent veto with $\sin^2 \frac{\theta_q}{2} = \frac{f_q(z_j)}{f^{\text{over}}(z_j)}$ for $|s\rangle = |q\rangle$ $(q = a, b)$

Two-flavor simulation with sampling

$t \in [t_{j+1}, t_j]$

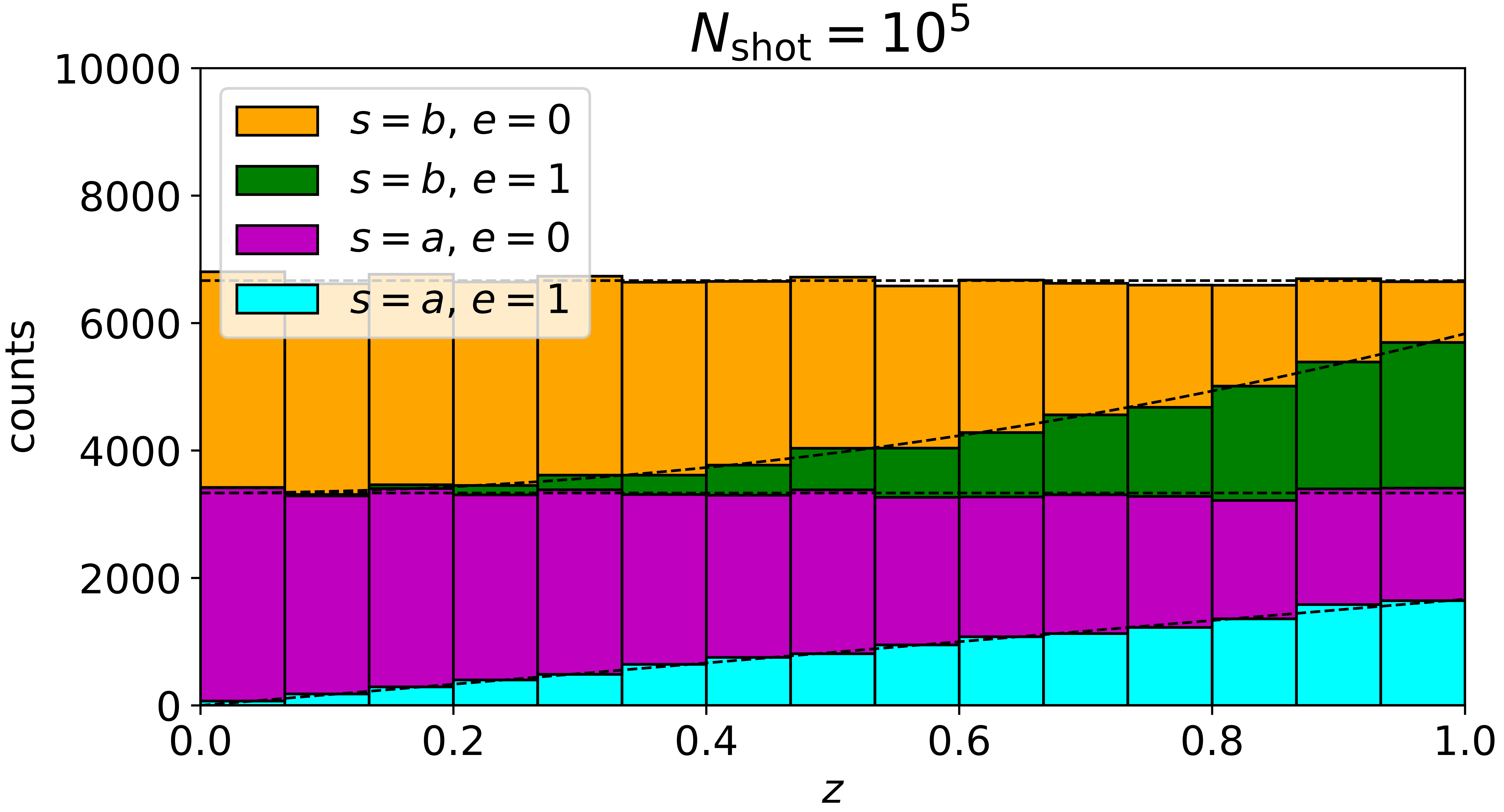


- Sample z according to over-estimated quantities with $f^{\text{over}}(z) \geq \max\left(f_a(z), f_b(z)\right)$

Veto procedure allows to use state-independent $\mathbf{f^{over}(z)}$ for sampling as far as $\mathbf{f^{over}(z)} \geq \max(\mathbf{f_a(z)}, \mathbf{f_b(z)})$ is available

- State-dependent veto with $\sin^2\frac{\theta_q}{2} = \frac{f_q(z_j)}{f^{\text{over}}(z_j)}$ for $|s\rangle = |q\rangle$ $(q = a, b)$

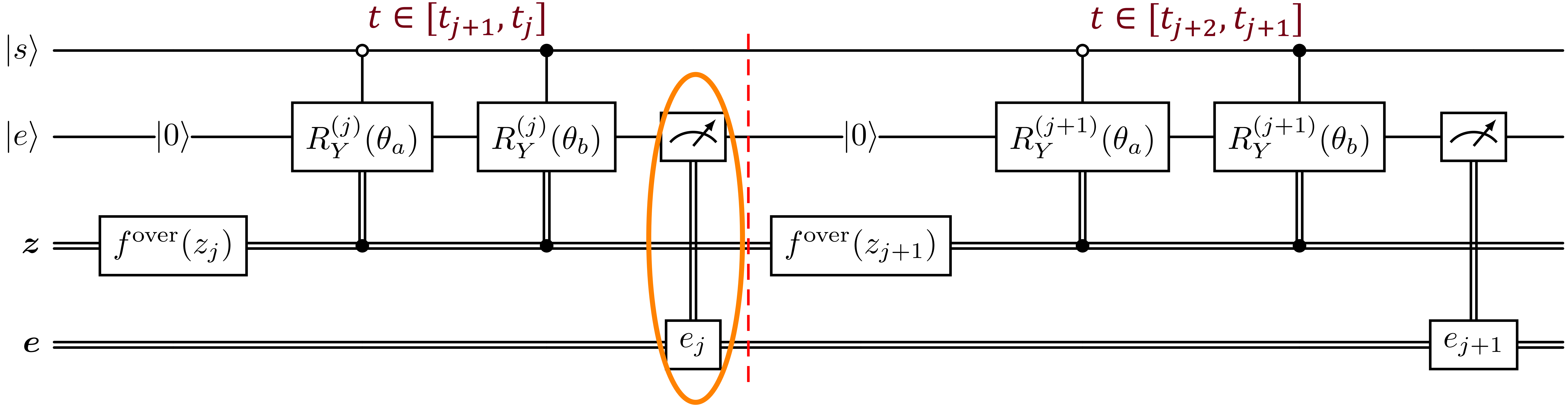
Numerical simulation by Qiskit



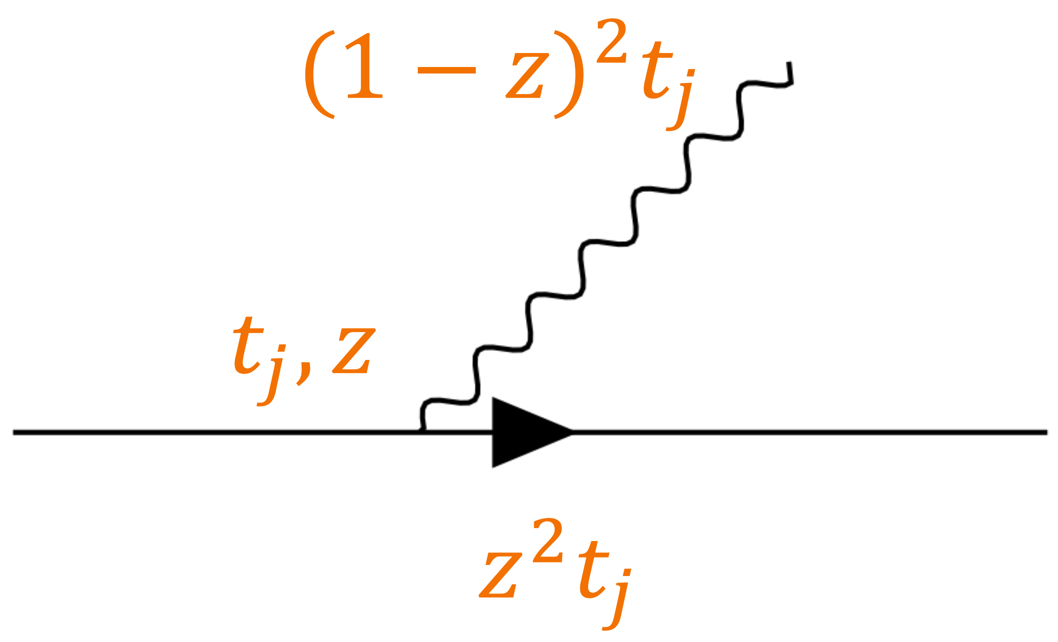
$f_a(z) = \frac{1}{2} z$, $f_b(z) = \frac{3}{4} z^2$ and $|s\rangle = \frac{1}{\sqrt{2}}(|a\rangle + |b\rangle)$

Multi-step simulation with kinematics

- N-step discretization $t_0 = E_0^2 > t_1 > t_2 > \cdots > t_N = \mu_{\mathrm{IR}}^2$



- Need mid-circuit measurement to track the preceding dynamics with full kinematics

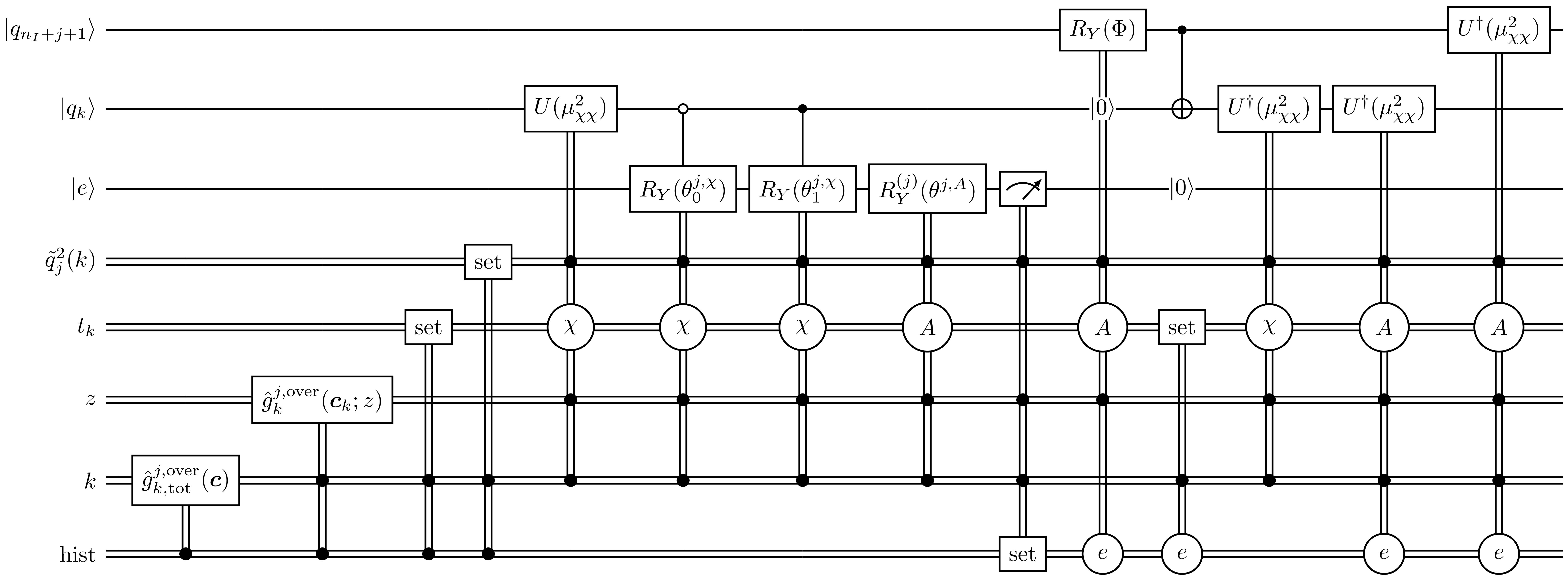


1) Add a parton
2) Virtuality jump

Quantum Veto Parton Shower

Bauer, **SC**, Yamazaki '24

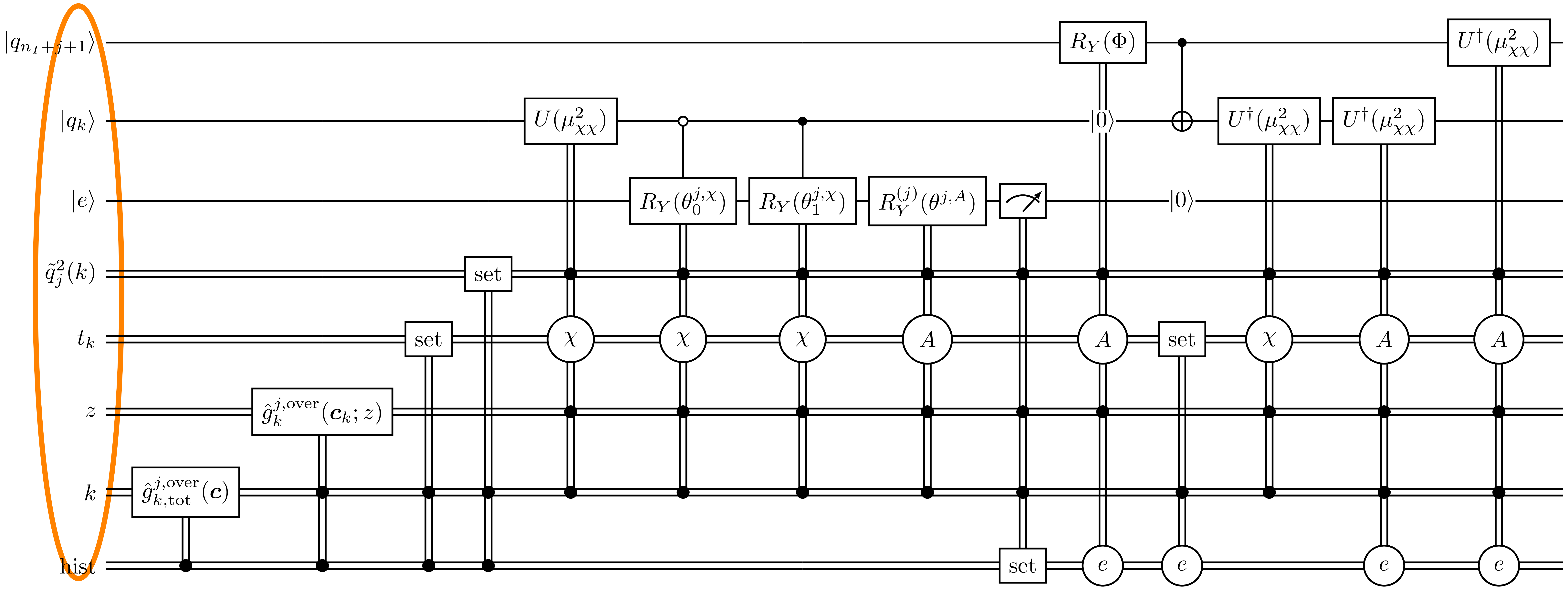
$t \in [t_{j+1}, t_j]$



Quantum Veto Parton Shower

Bauer, **SC**, Yamazaki '24

$t \in [t_{j+1}, t_j]$



❖ Particle register $|q_k\rangle$ for each parton k stores fermion flavors in $\lceil \log_2 N_f \rceil$ qubits

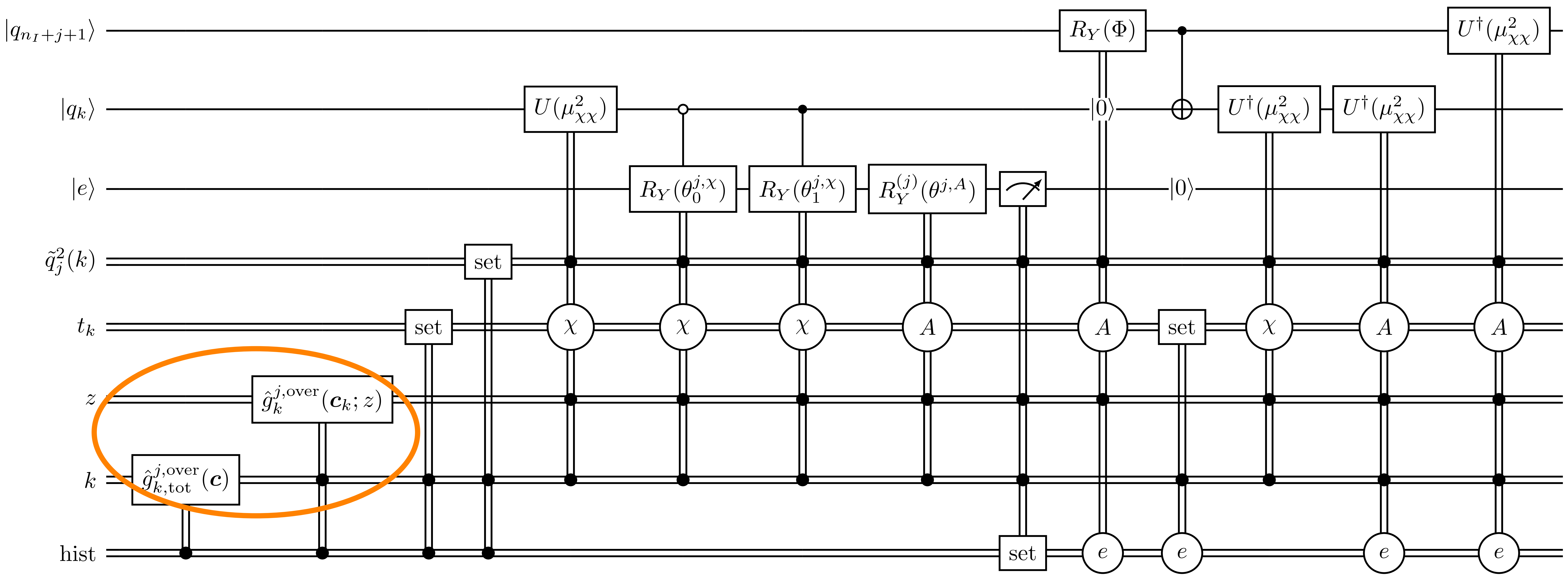
❖ Virtuality of each patron $\tilde{q}_j{}^2(k)$, whether it is a fermion / gauge boson, stored in classical bits

❖ Emission history is also stored in classical bits

Quantum Veto Parton Shower

Bauer, **SC**, Yamazaki '24

$t \in [t_{j+1}, t_j]$



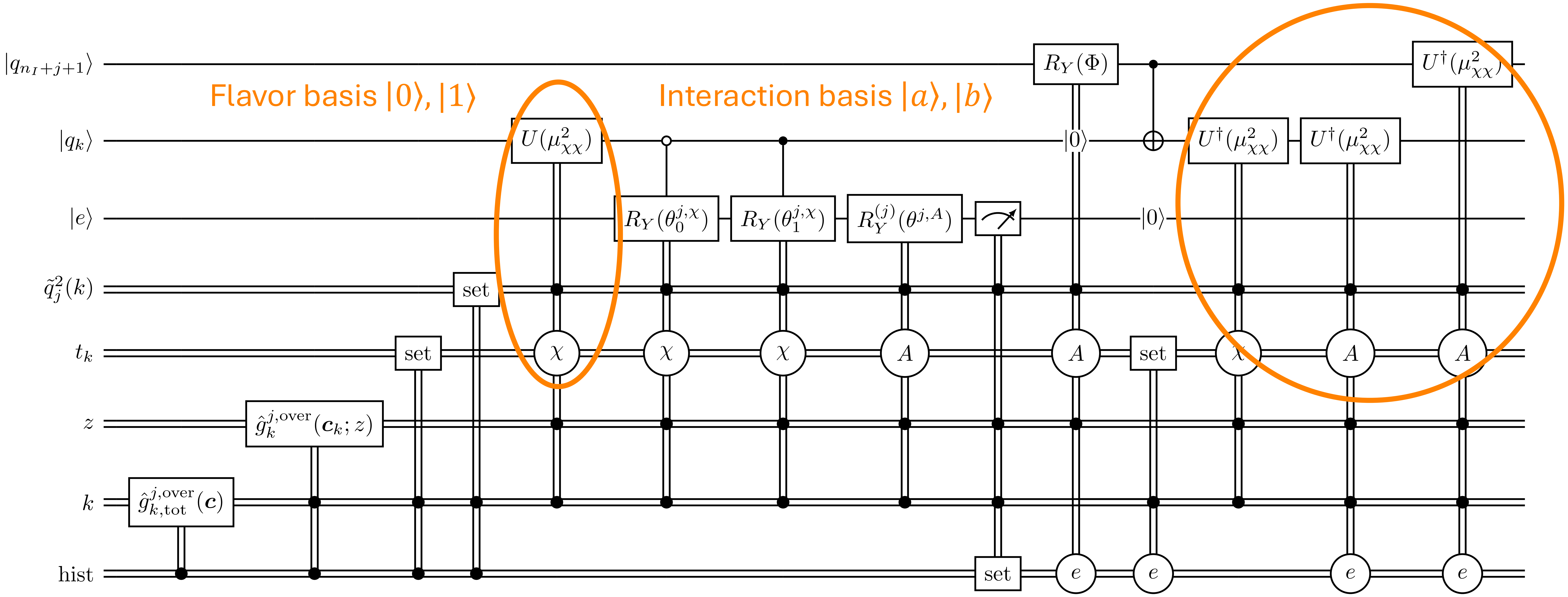
❖ Sampling of k (a candidate parton that undergoes emission) and z (energy fraction)

- Sampling of k can be done classically again thanks to the over-estimated quantities
- Candidate splitting topology and kinematics is fixed

Quantum Veto Parton Shower

Bauer, **SC**, Yamazaki '24

$t \in [t_{j+1}, t_j]$



❖ Basis rotation of fermion (if necessary)

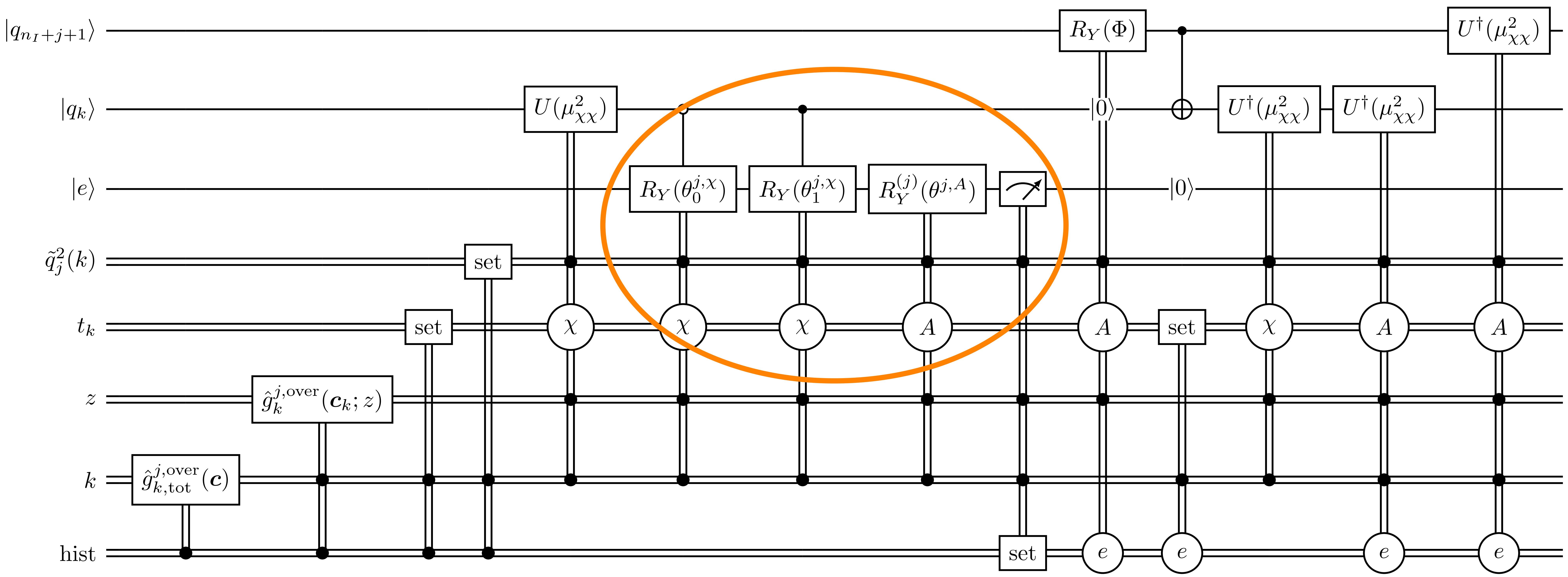
- Due to the RGE flow, the rotation angle is scale/kinematics-dependent
- Suitable choice of the RG scale is process dependent

Herwig++ Physics and Manual [0803.0883]

Quantum Veto Parton Shower

Bauer, **SC**, Yamazaki '24

$t \in [t_{j+1}, t_j]$

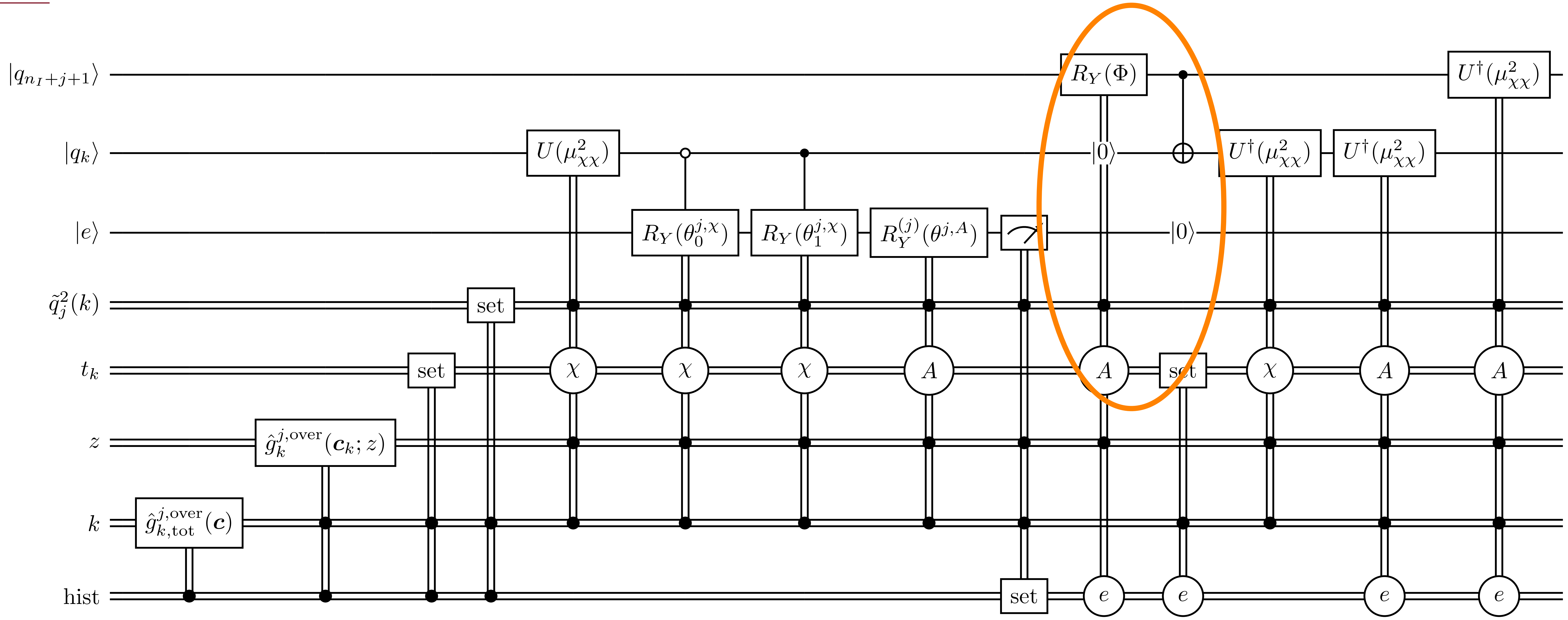


- Veto and determine whether the emission occurs through the mid-circuit measurement

Quantum Veto Parton Shower

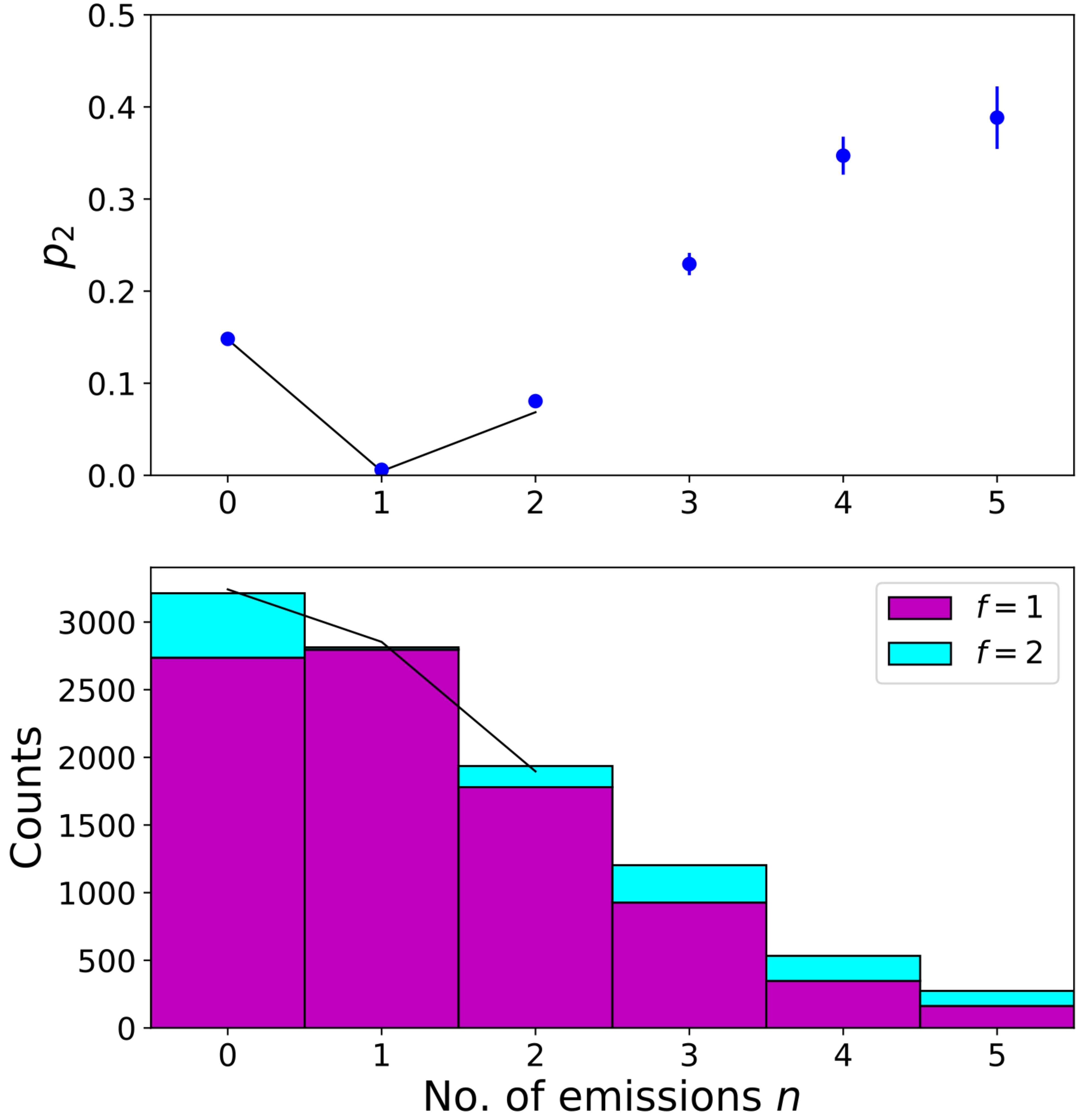
Bauer, **SC**, Yamazaki '24

$t \in [t_{j+1}, t_j]$



- ❖ If emission occurs, state update is necessary
  - k = fermion, add a new gauge boson
  - k = gauge boson, generate an **entangled** state $|q_k\rangle|q_{\mathrm{new}}\rangle = \frac{1}{\alpha_a^2+\alpha_b^2}(\alpha_a|a\rangle|a\rangle + \alpha_b|b\rangle|b\rangle)$

Numerical results of QVPS



Bauer, **SC**, Yamazaki ‘24

Future directions

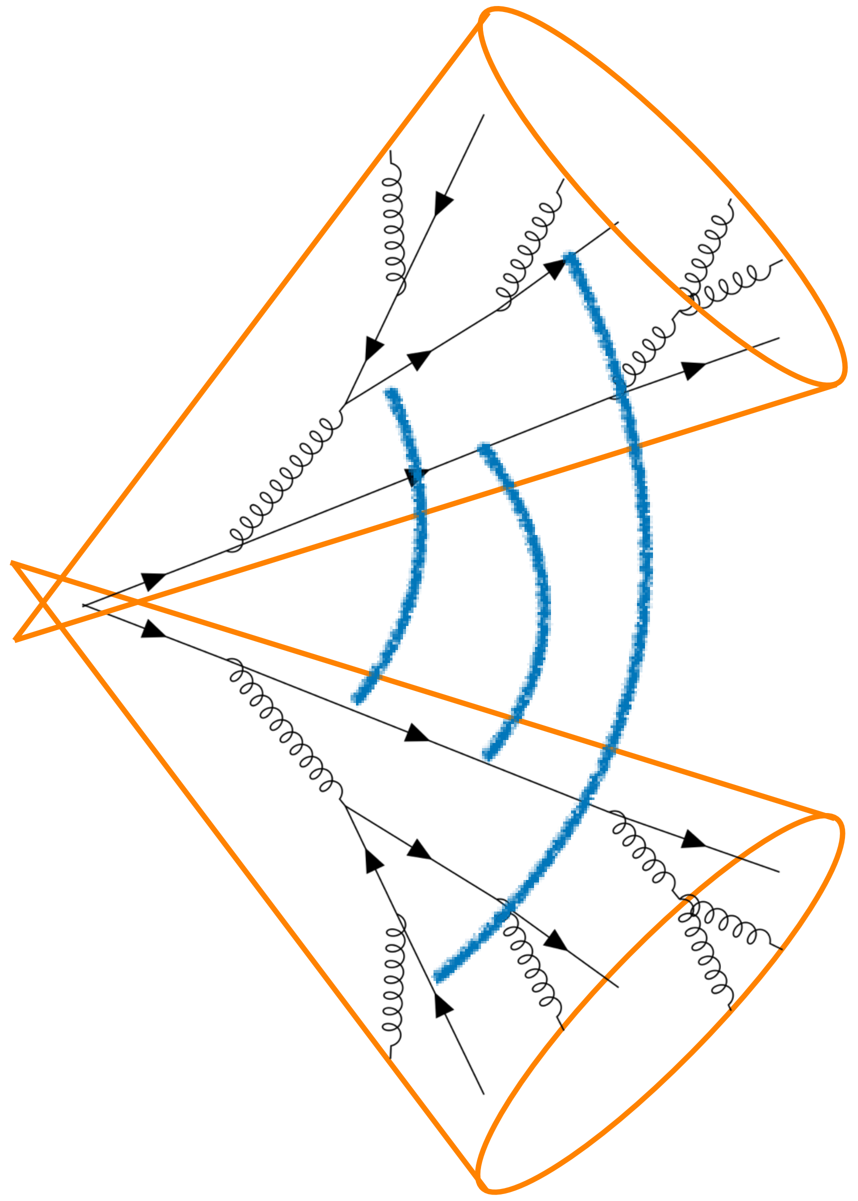
❖ Construct more efficient algorithms

- $\Delta\mathcal{P}_a$, $\Delta\mathcal{P}_b \ll 1$ enforces fine mesh of t
- Directly sampling t with veto
- Gate cost $O(N) \to O(\langle n \rangle)$

❖ Exclusive observables & soft logs Work in progress

- Spin interference
- Color interference

❖ Next-to-leading logarithms

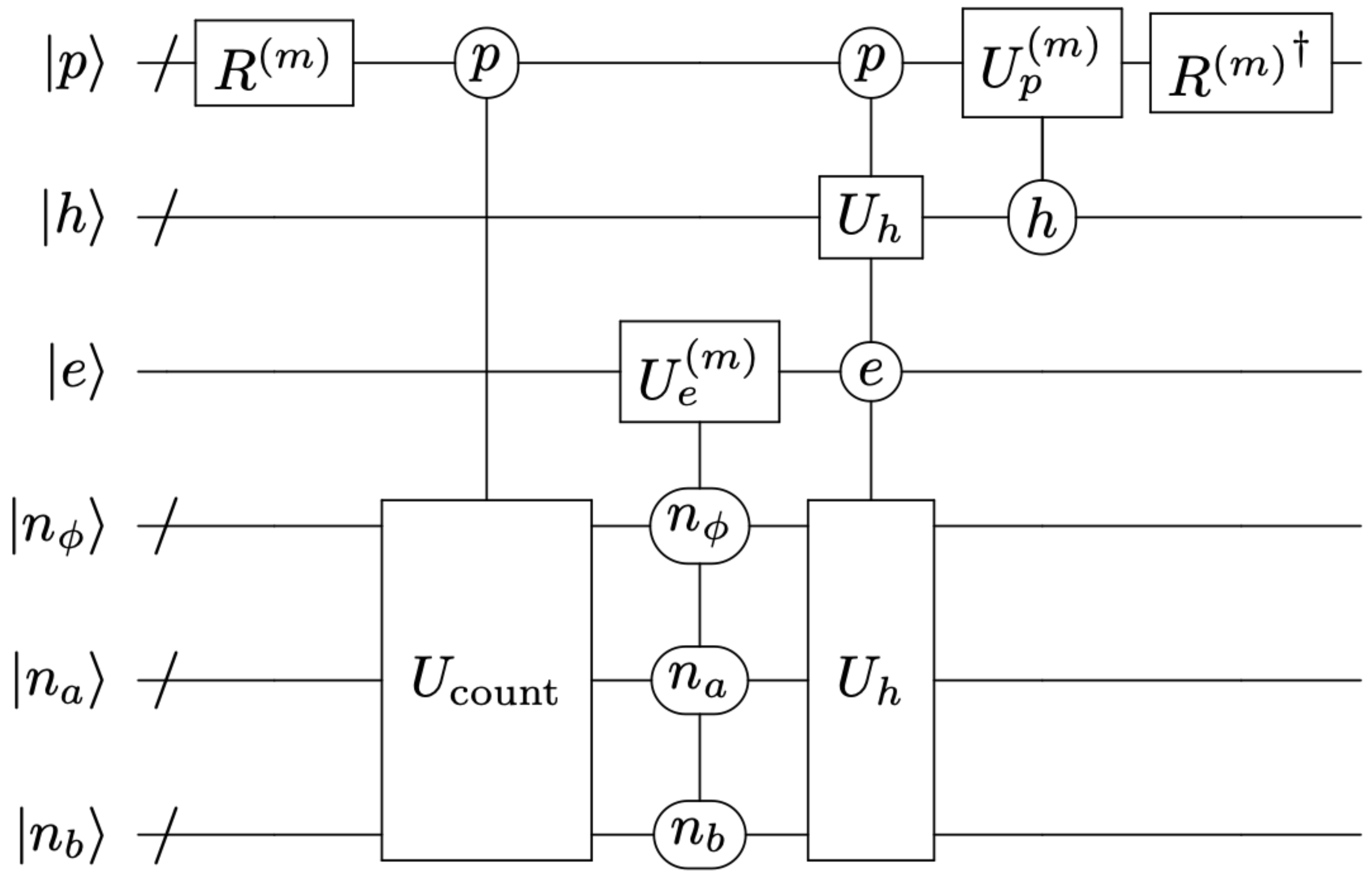


- Quantum resources required

Register	Purpose	2 flavors
$\lvert s\rangle$	Particle state	$(N+n_I)\log N_f$
$\lvert e\rangle$	Did emission happen?	1

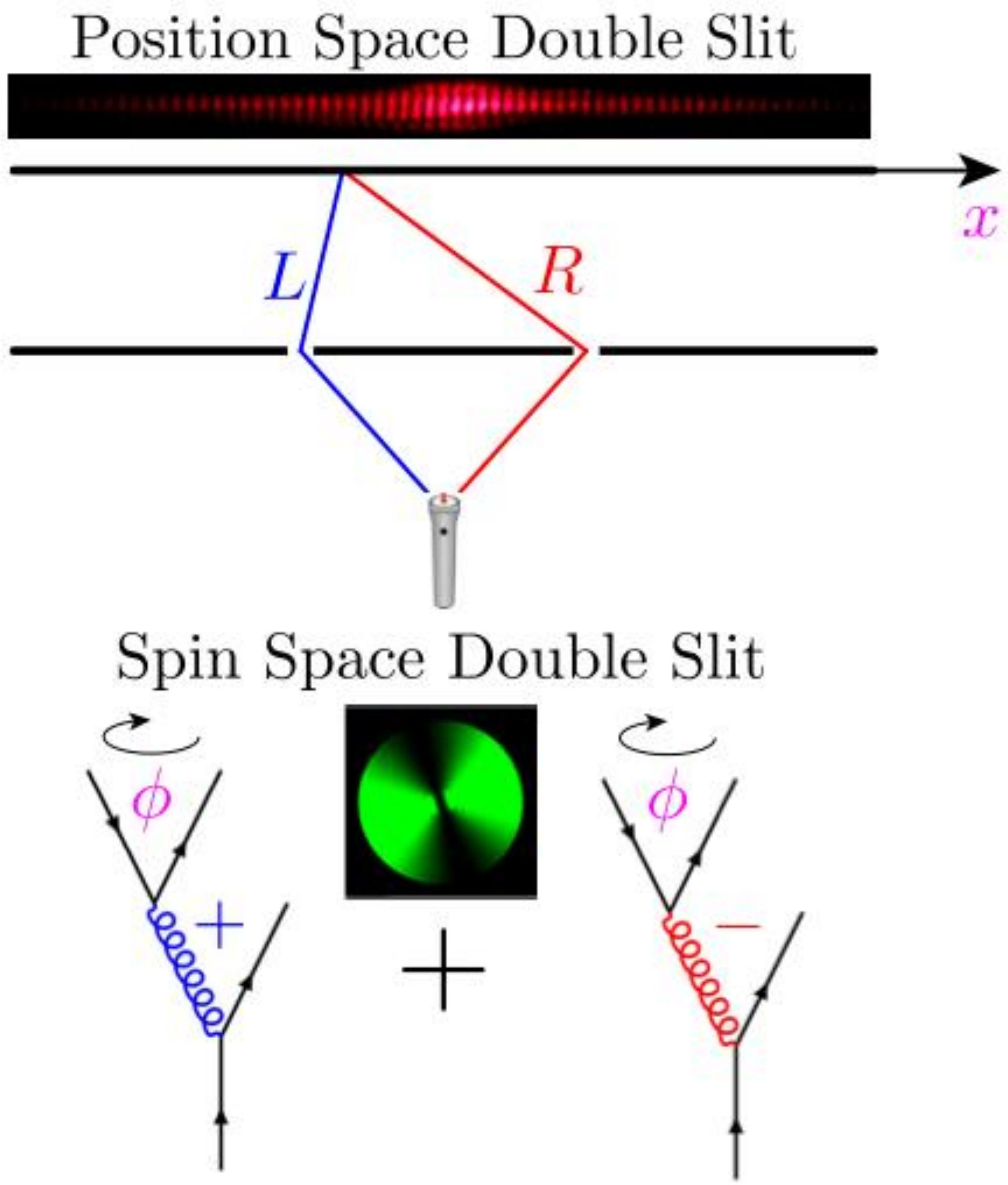
Element	Purpose	Gate costs
$U(\mu^2)$	Flavor rotation	$N_f^2 N$
$R_Y(\theta)$	Emission	$N_f N$
$R(\Phi)$	Particle update	$N_f^2 N$

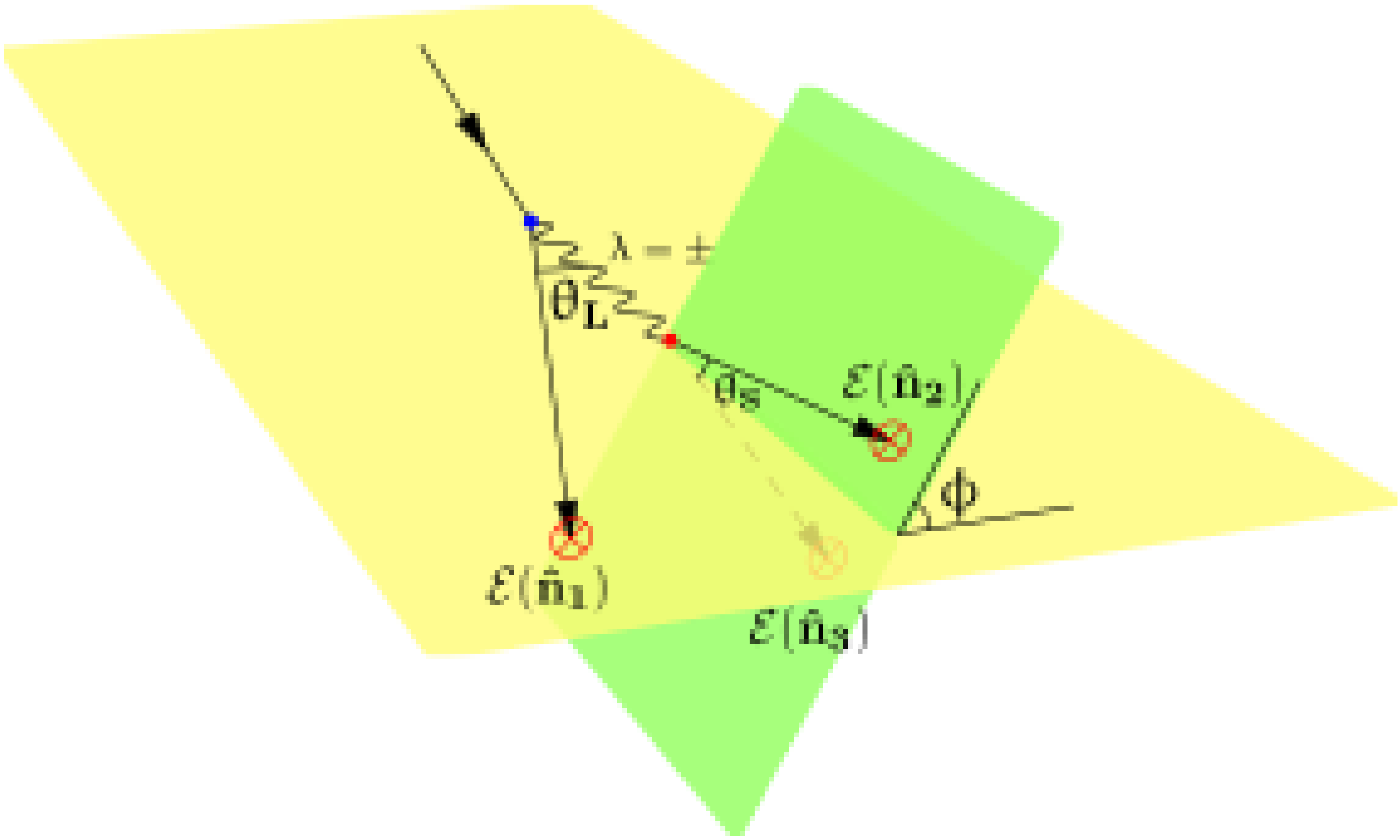
- Emission history in a qubit register



C. W. Bauer, et al. [1904.03196]

Spin interference



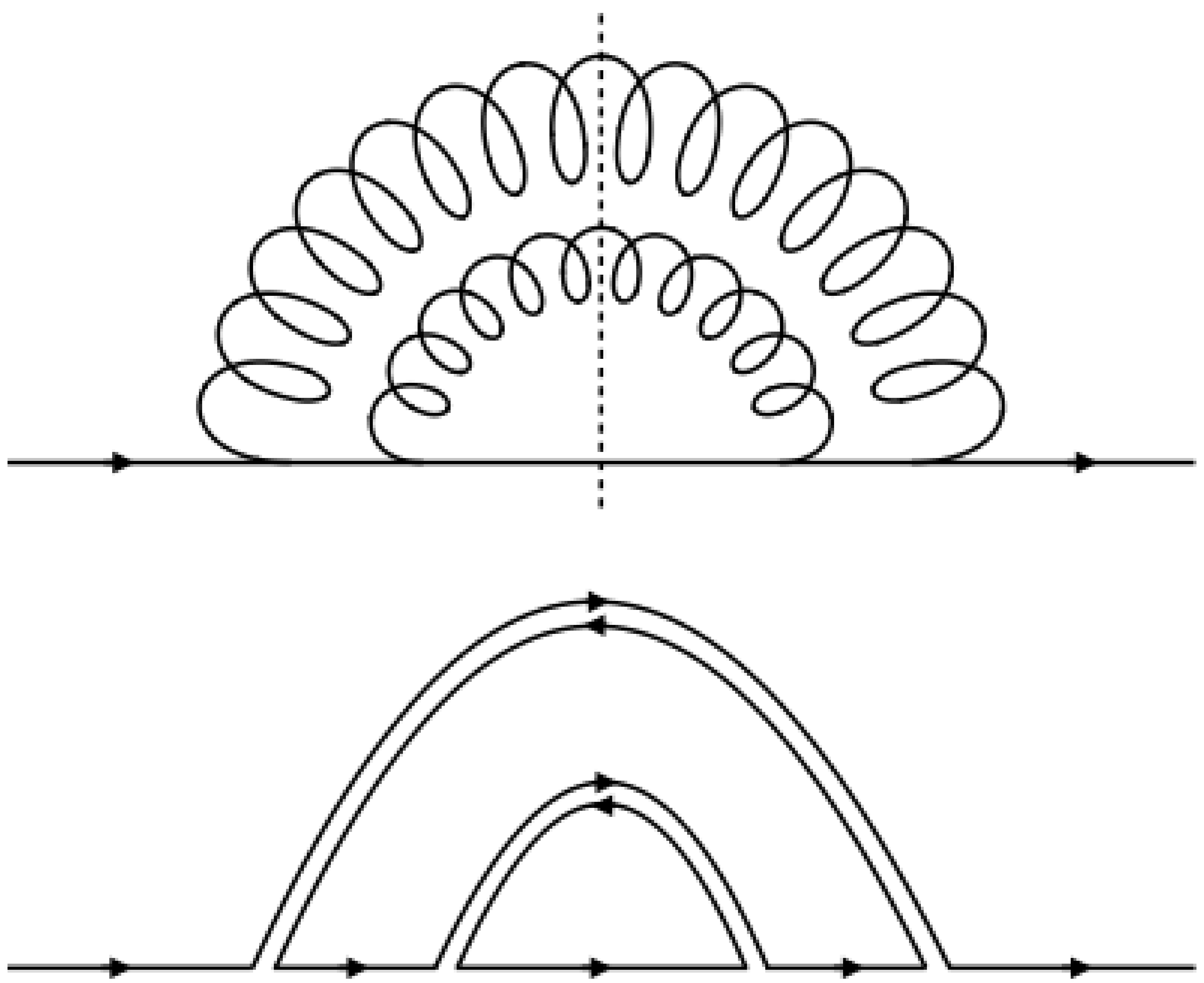


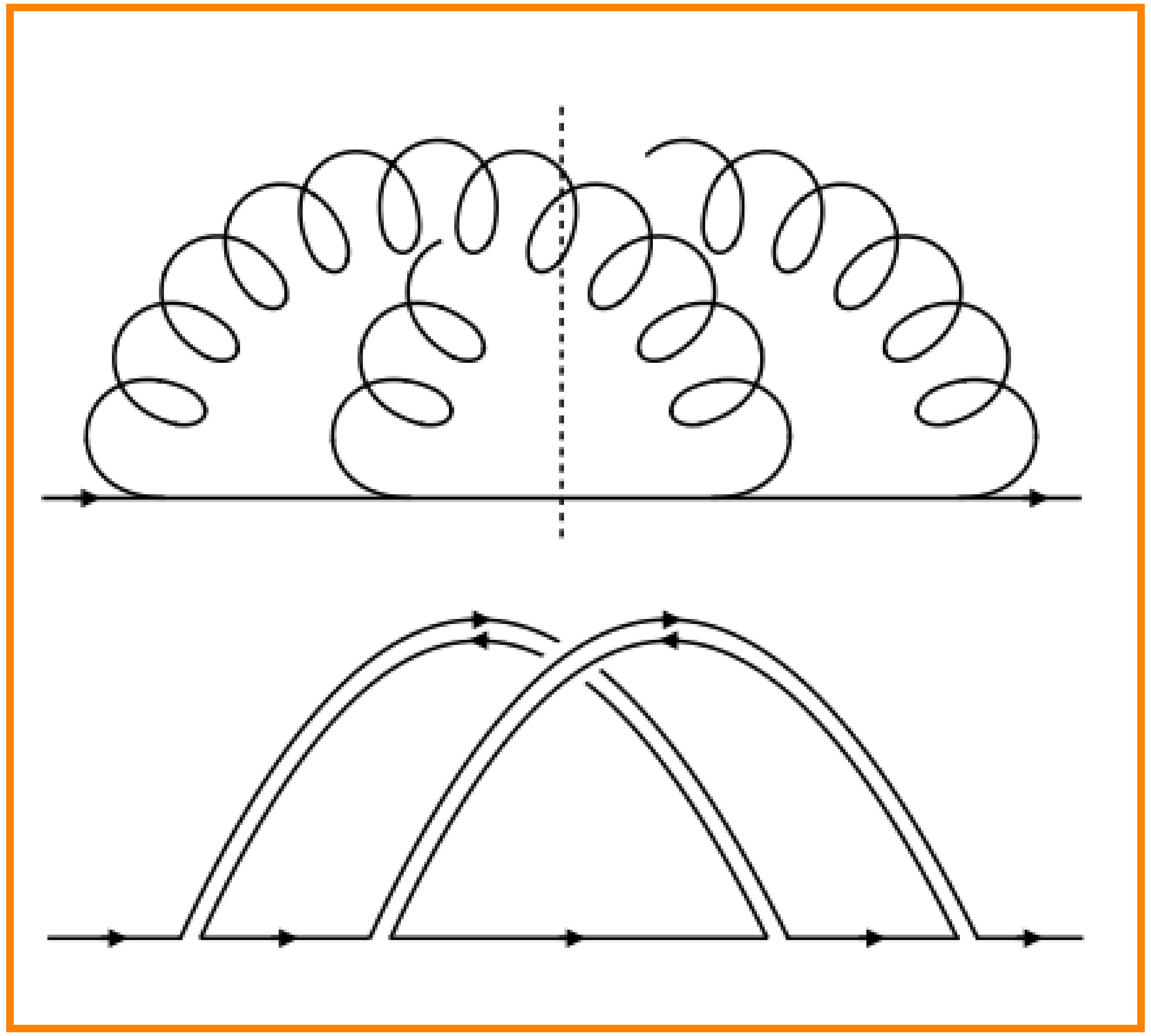
H. Chen, et al. [2011.02492]

- Exclusive observables as a function of ϕ can exhibit spin interference
- Visible in the squeezed limit of 3-point energy correlators

Color interference

$O(1/N_c^2)$





- $\rho_{\text{QCD}} = \frac{1}{2}|g_1 g_2\rangle\langle g_1 g_2| + \frac{1}{2}|g_2 g_1\rangle\langle g_2 g_1| + \frac{2C_F - C_A}{4C_F}\left(|g_1 g_2\rangle\langle g_2 g_1| + |g_2 g_1\rangle\langle g_1 g_2|\right)$
- Visible in 3-point (particle ID-ed) energy correlators

A. J. Larkoski [2205.12375]

Conclusion

Problem

- A non-trivial flavor structure revives interference effects at the LL-level
- Cannot be tracked with the classical parton shower

What we did

- Constructed the quantum veto parton shower (QVPS)
- Demonstrated the phenomenological implications

Future directions

- Soft interference
- Color interference
- Physics case studies and further extensions of circuit

Application to QFT

- Quantum simulation of parton shower — C. W. Bauer+ [1904.03196]
  - Can naturally capture the quantum nature of phenomena
- S-matrix calculation of lattice gauge theory
  - (# of qubits required) $\sim \log$ (# of classical d.o.f.s) — C. W. Bauer+ [2102.05044]
- Screening/confinement in Schwinger model with a topological term
  - Map between spin-/fermion-/boson-systems e.g.) Jordan-Wigner transformation

  M. Honda, E. Itou, Y. Kikuchi, L. Nagano, T. Okuda [2105.03276]
- Event Classification with Quantum Machine Learning

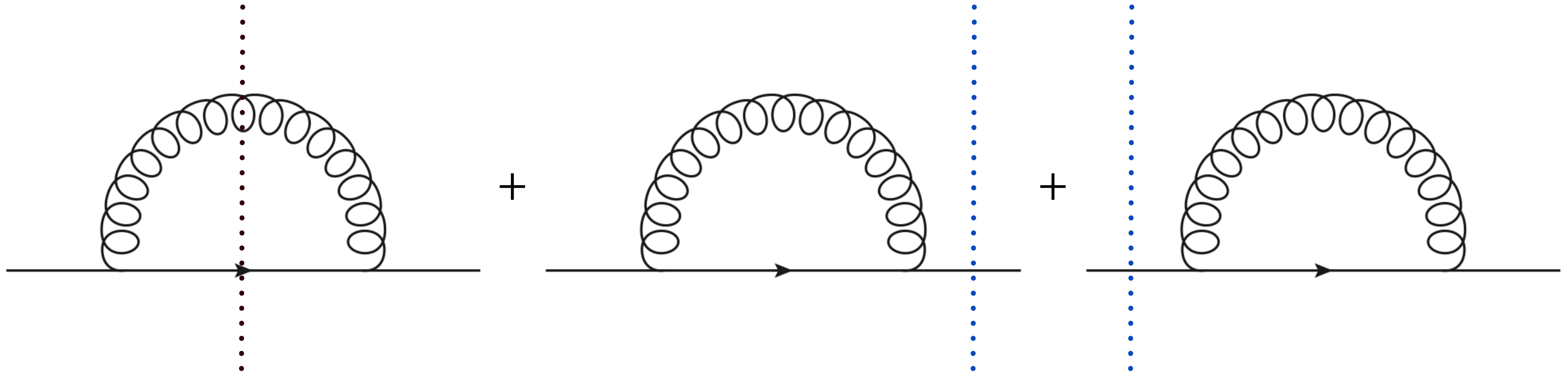
  K. Terashi, M. Kaneda, T. Kishimoto, M. Saito, R. Sawada, J. Tanaka [2002.09935]

Unitarity

- So far focused on tree-level processes

$$\sigma^{\text{incl.}} \supset \sigma_0^{\text{FO}}[1 \quad + \quad \alpha_s \ln^2 \quad + \quad (\alpha_s \ln^2)^2 \quad + \cdots]$$

- Unitarity ensures order-by-order cancellation of IR singularity

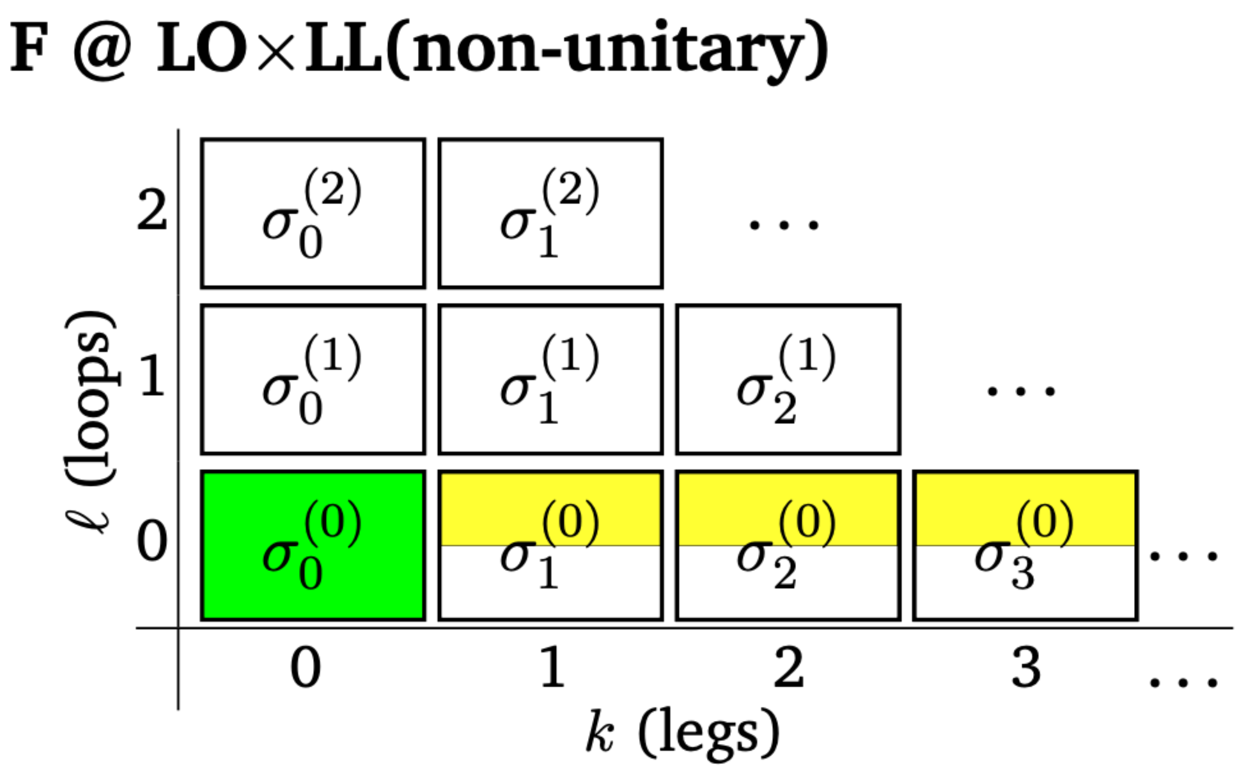


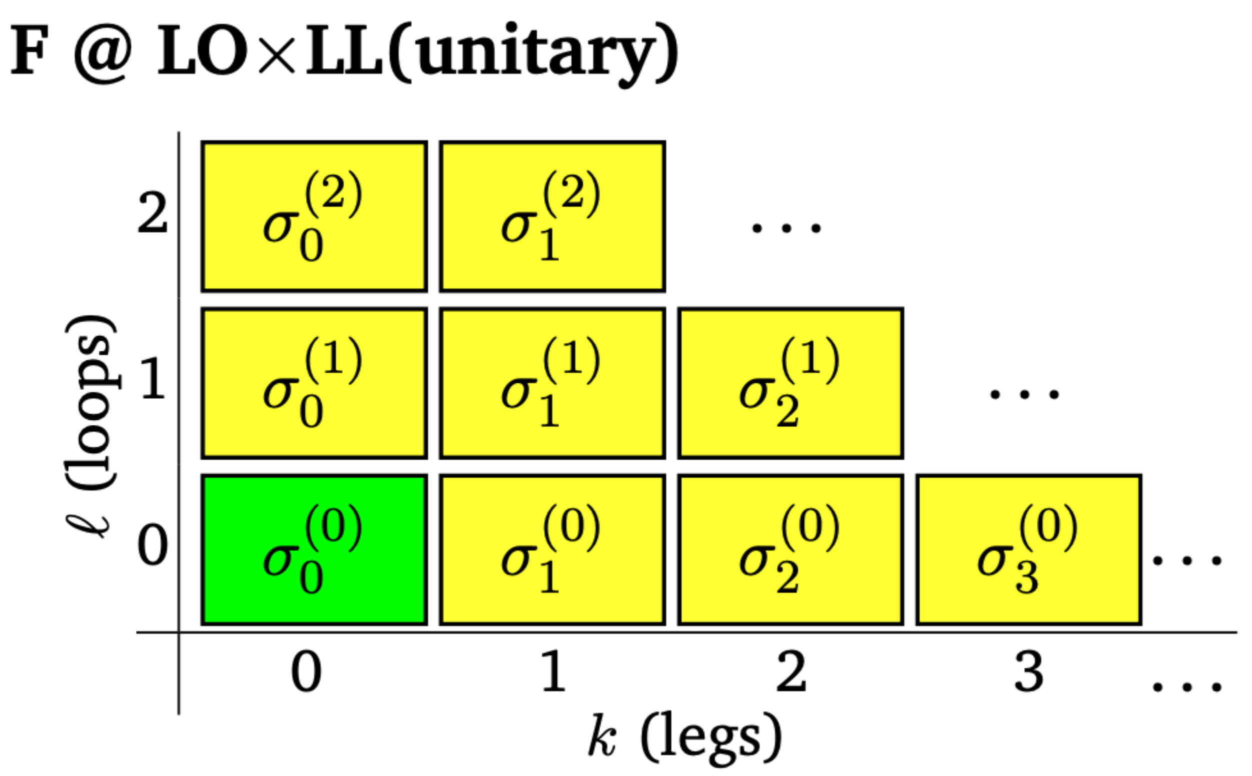
= IR finite

$$\sigma_0^{\text{FO}} C_F \frac{\alpha_s}{2\pi} \ln^2 \frac{E_0^2}{\mu_{\text{IR}}^2} \qquad -\sigma_0^{\text{FO}} C_F \frac{\alpha_s}{2\pi} \ln^2 \frac{E_0^2}{\mu_{\text{IR}}^2}$$

- $\sigma^{\text{incl.}} = \sigma_0^{\text{FO}}$
- $\sigma_{n \geq k}^{\text{incl.}} = \sigma_k^{\text{tree,LL}}$

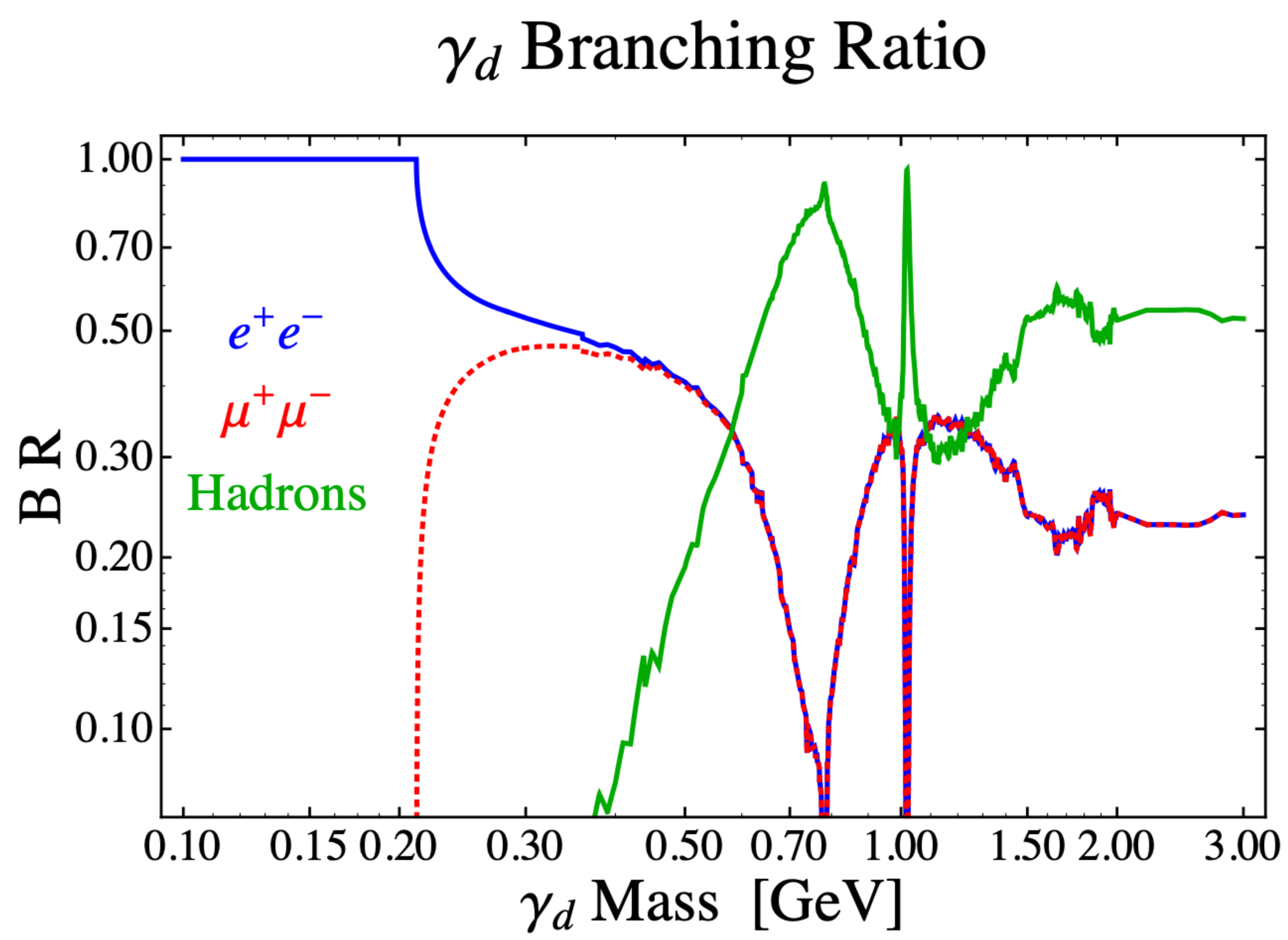
P. Skands '12 "Introduction to QCD"





Phenomenology example: lepton jets

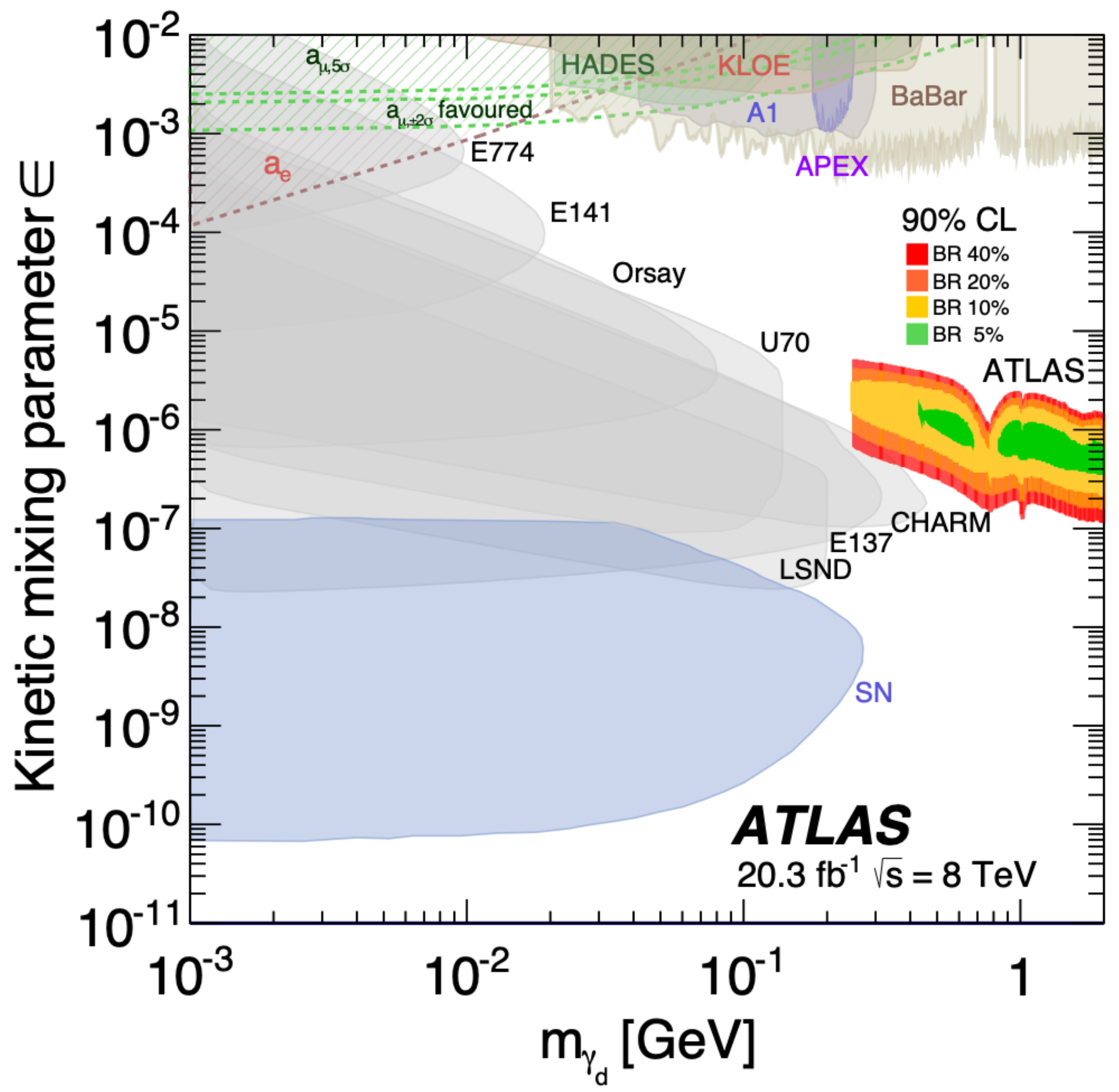
- Observe A' decay products from $pp \to \bar{\chi}\chi + nA'$
  - A' decay through kinetic mixing



A. Falkowski[+] [1002.2952]

- "Lepton jets" for $m_{A'} \lesssim \mathrm{GeV}$

C. Cheung[+] '09, P. Meade[+] '09, A. Falkowski[+] '10



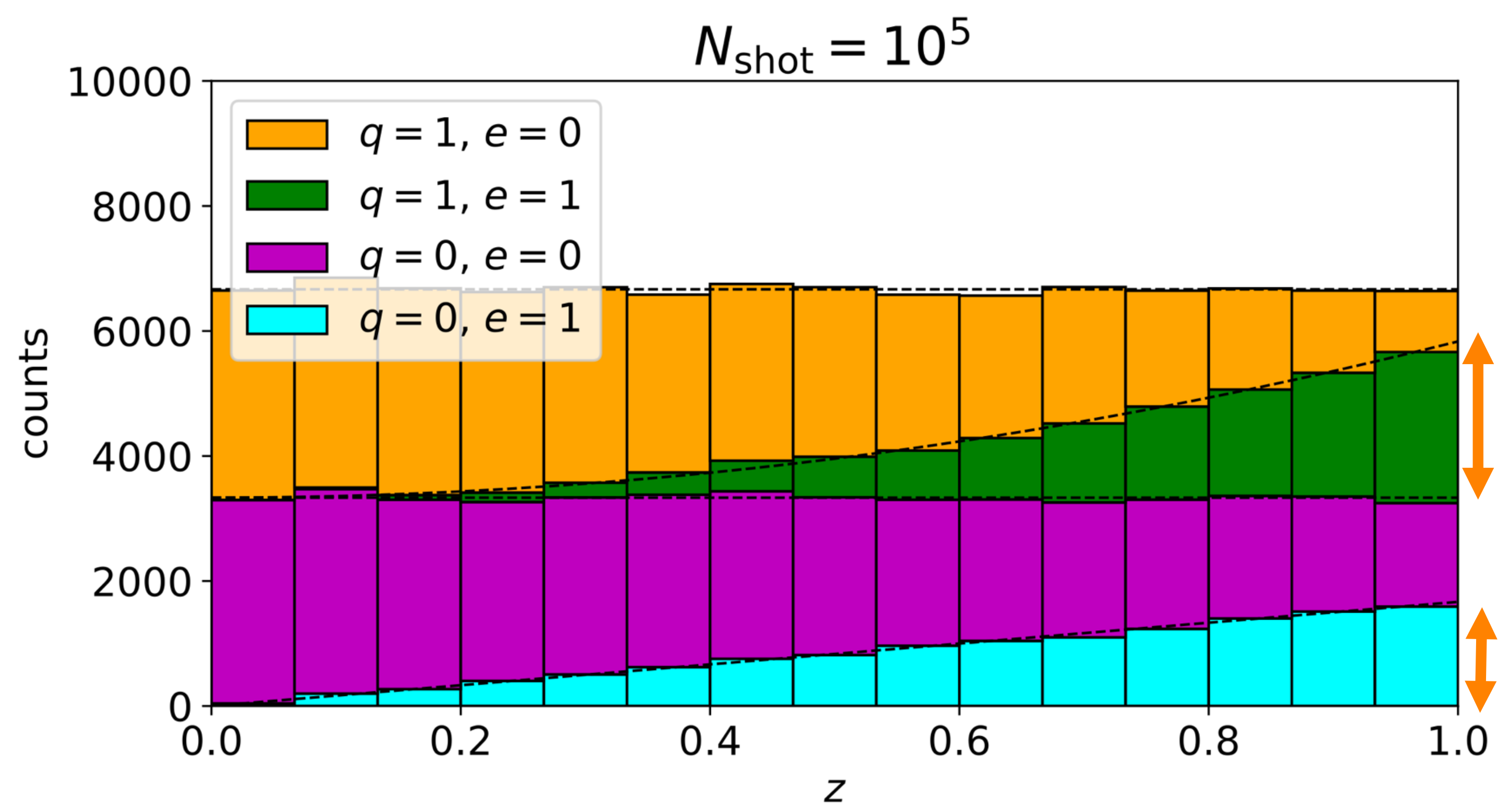
ATLAS [1212.5409]
ATLAS [1409.0746]

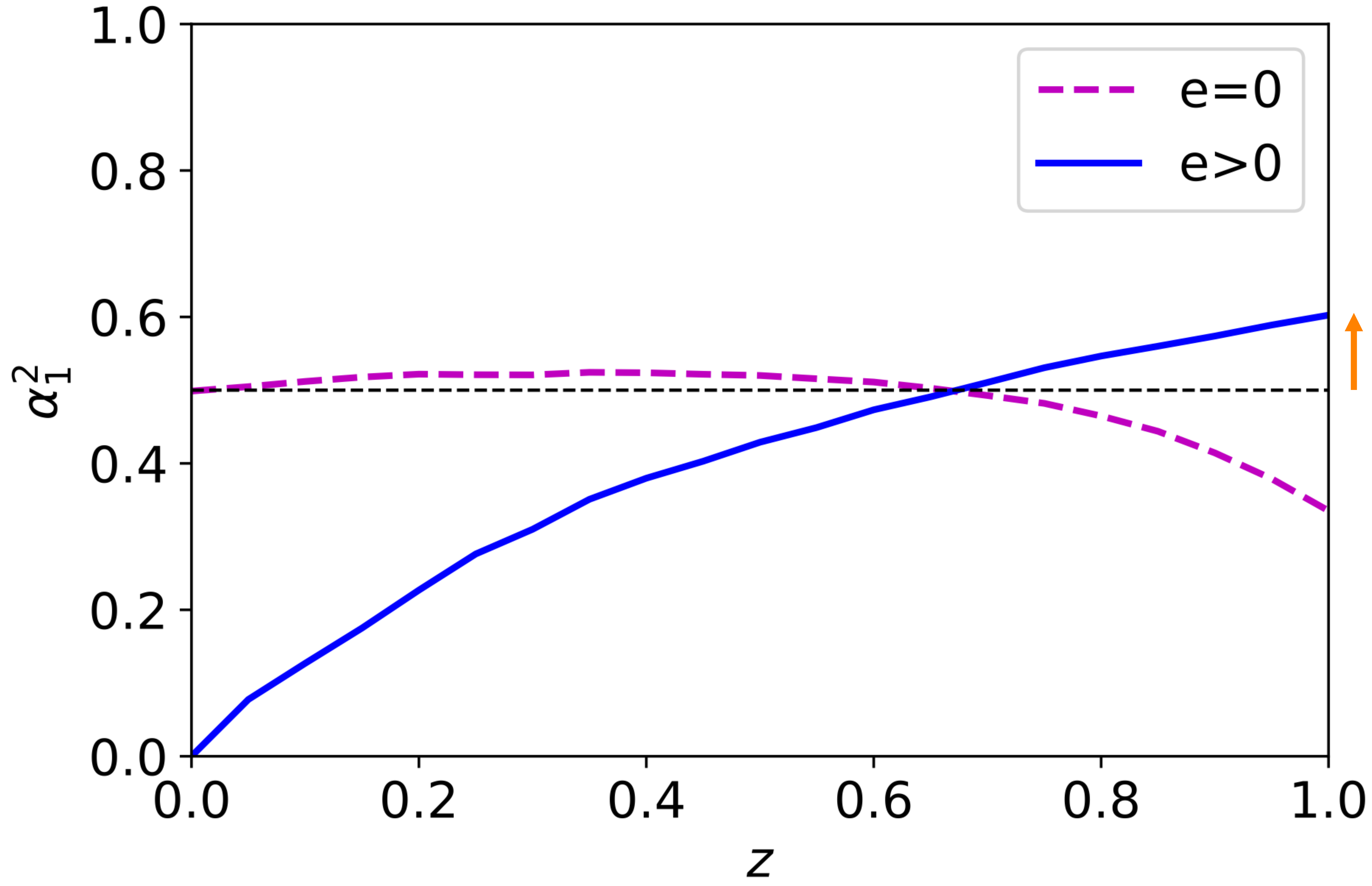
- Cuts on lepton multiplicity eg) $\geq$ 4 muons

- Interference effect on number distribution of emissions matters

Numerical simulation by Qiskit

- ❖ $|s\rangle = \alpha_0|a\rangle + \alpha_1|b\rangle$ after meas. of $|e\rangle$

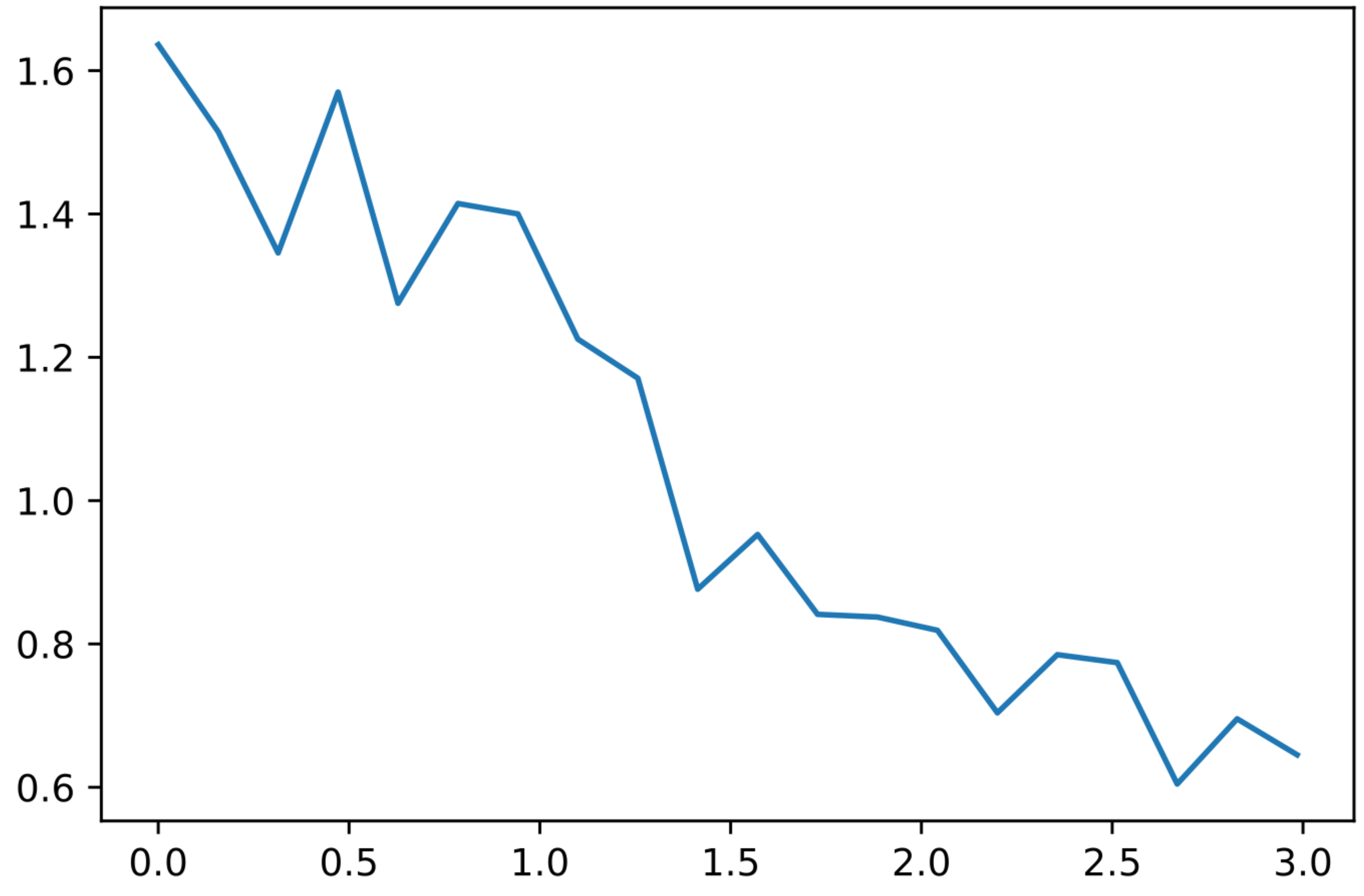




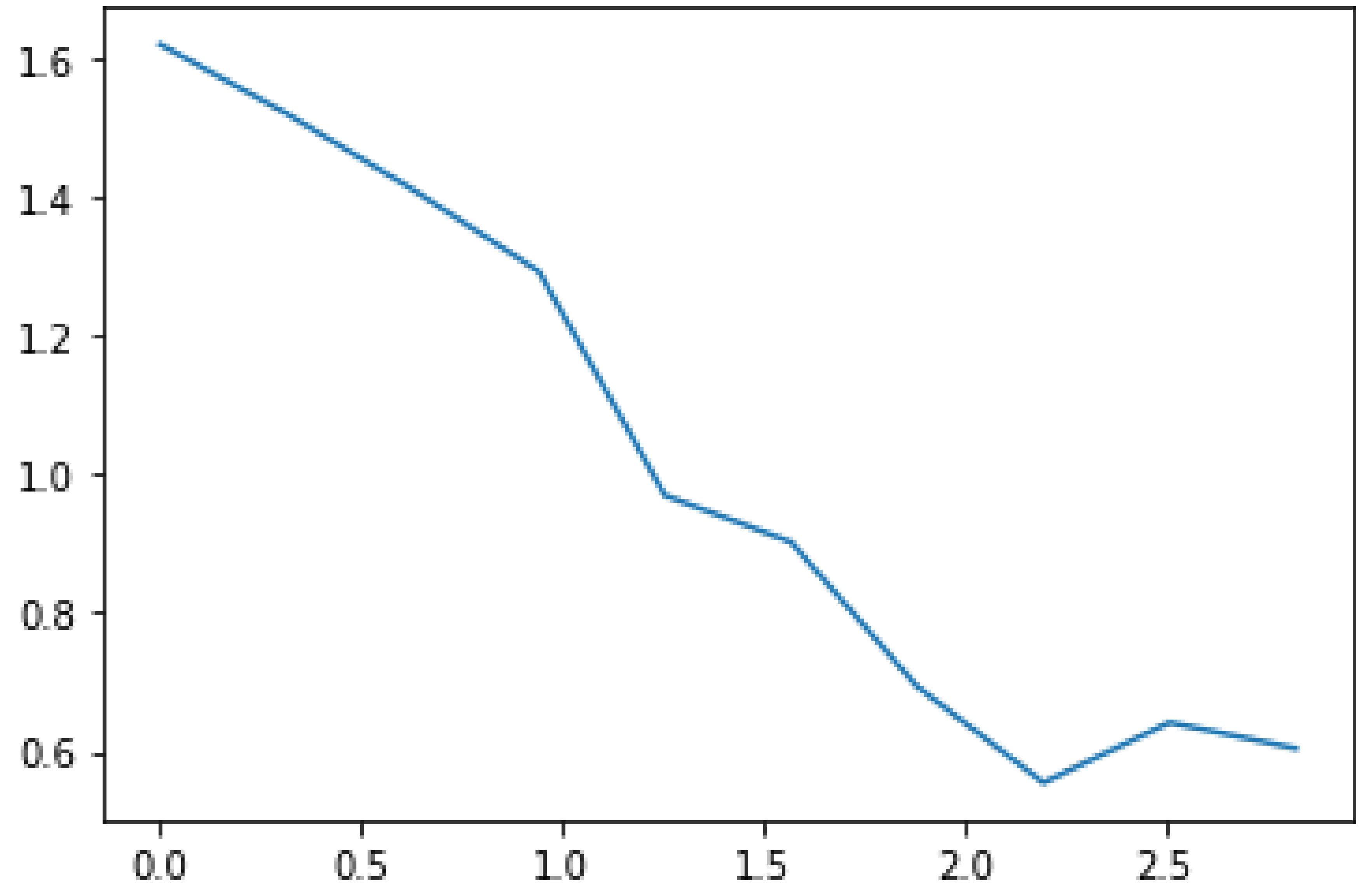
- ❖ Analytically / Numerically checked quantum state evolution is OK up to $O\left(\Delta\mathcal{P}_q^2\right)$
  - Require $\Delta\mathcal{P}_a$, $\Delta\mathcal{P}_b \ll 1$

Error mitigation

- Large $N \gtrsim \mathcal{O}(30)$ w/o mid-circuit measurement requires real quantum computers



IBMQ_Santiago (5 qubits) w/o error mitigation

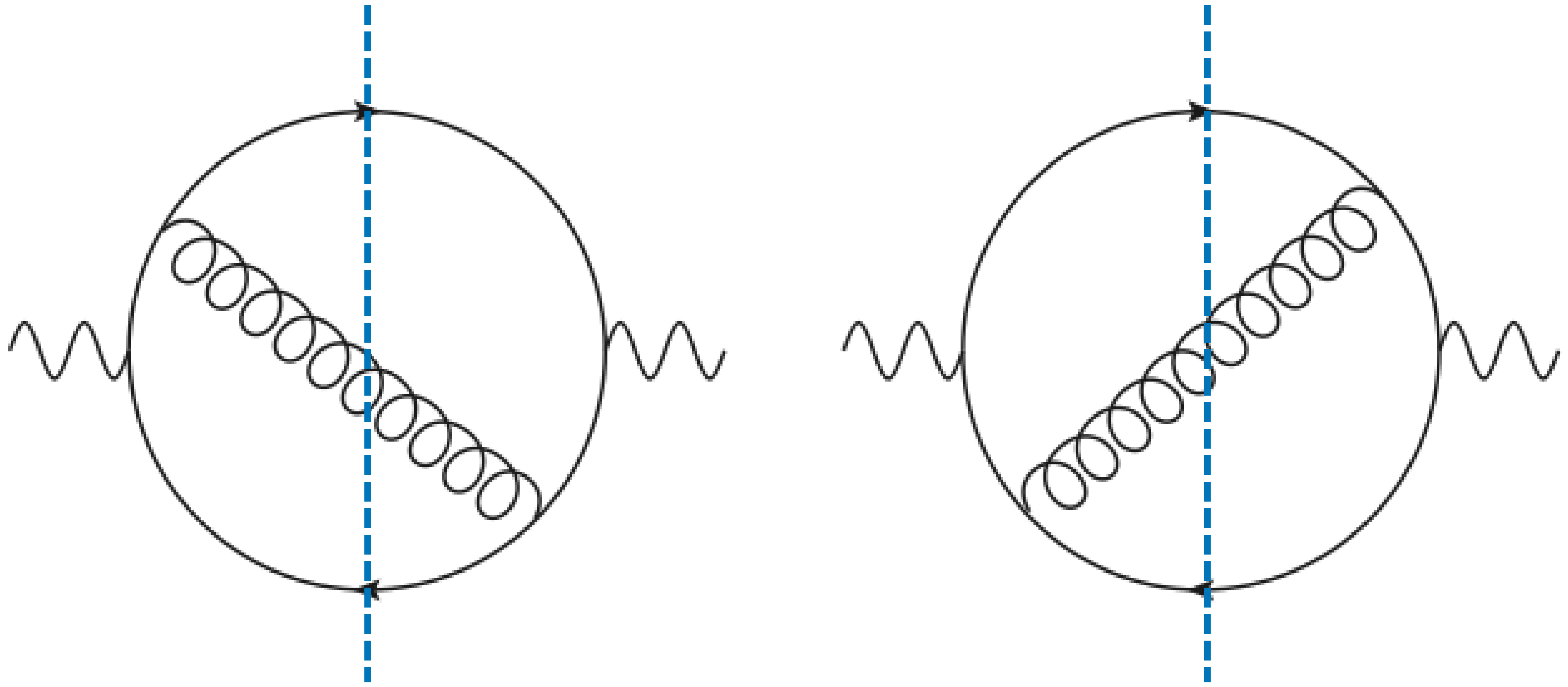


after error mitigation

- Fight with noise in quantum computation
  - Error correction
  - Resolve the reason of machine dependence

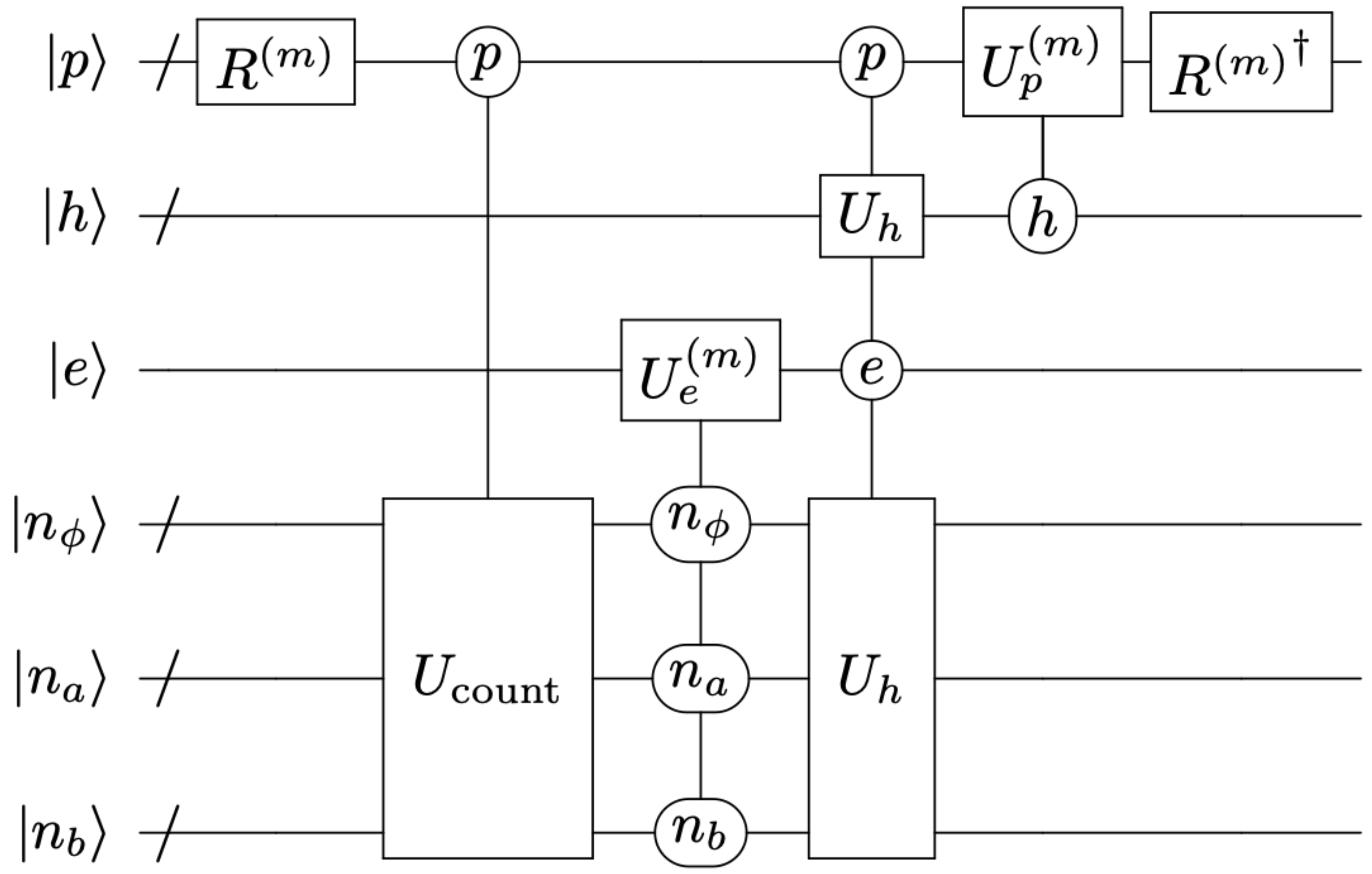
Quantum simulation of soft interference

- Ex) $q\bar{q} + g$



- No collinear logs
  - In different jet cones
- Soft logs
  - Wide-angle soft emissions

- Interference of emission histories
  - Need to extend our approach with quantum history registers



C. W. Bauer, et al. [1904.03196]